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CALIBRATION APPROACH FOR NON RESPONSE ADJUSTMENT UNDER RANKED SET SAMPLING

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ABSTRACT

Non-response poses a significant concern within survey sampling, occurring when individuals are either unavailable or unwilling to respond to survey questions. To address this issue, the application of non-response adjustment through auxiliary variables is widely employed to estimate missing data points. This study delves into the extensive possibilities presented by this approach, particularly focusing on calibration weighting. The study specifically investigates the challenge of non-response within the context of ranked set sampling (RSS).

Within this study, we introduce two novel calibration estimators for population means. One estimator addresses scenarios of complete response, while the other pertains to cases of non-response under the ranked set sampling technique. To evaluate their performance, the Mean Square Error (MSE) and Bias expressions are derived. Furthermore, the performance of the proposed estimators is assessed through comprehensive simulation studies, encompassing both artificial and real-world datasets. For this purpose, the Mean Square Error and Bias for the ranked set samples of sizes 5, 8, 10, and 25, encompassing a total of 10,000 samples for each size category are calculated.

Our findings reveal that the proposed calibration estimator, along with the imputation through calibration for non-response, consistently generates more efficient estimates compared to mean and median imputation techniques. This underscores the potential of the calibration approach in mitigating the impact of non-response, offering improved accuracy and robustness in survey sampling.

KEYWORDS

Rank Set Sampling, Mean Square Error, Non-response, Calibration.

1. INTRODUCTION

The ranked set sampling (RSS) approach was proposed by McIntyre in 1952 for the cases when taking the actual measurements for sampling units was difficult but ranking a set of sample units either informally or formally was relatively easy and reliable. The Ranked Set Sampling (RSS) technique gives more structure to the sample items and increases the amount of information contained in the sample. This extra structure and information provided by the judgment ranking enables RSS to be more efficient than

Simple Random Sampling (SRS) with the same number of observations (Wolfe, 2004, 2012).

Let $Y_{(i:n)}$ denotes the i^{th} order statistic from the set of n values observed for $i=1,2,.....n$ observations and $Y(i:n)j$ denote the ith order statistic in the sample of size n for jth sample ($j=1,2,...m$). The n measurements $Y_{(1:n)j},...,Y_{(n:n)j}$ are the order statistics. However, unlike order statistics, ranked set sampling treats these ordered values as distinct and independent observation units, with each value contributing unique insights about different characteristics of the population. In this approach, the joint probability density function for the ordered values is expressed as follows:

$$grss\left(y_{(1:n)j},...,y_{(n:n)j}\right)=\prod_{i=1}^{n}\int(i)\left(y_{(i:n)}\right)]$$

where

$$\int(i)\left(y_{(i:n)}\right)=\frac{k!}{(i-1)!(k-i)!}.\left[F\left(y_{(i:n)j}\right)\right]^{i-1}\left[1-F\left(y_{(i:n)j}\right)\right]^{k-i}\int\left(y_{(i:n)j}\right)$$

A RSS of size n is very different than a simple random sample (SRS) of the same size in many ways. A SRS is selected in such a way that the n units in the sample are independently and identically distributed whereas in RSS the units are considered mutually independent but are not evenly distributed.

In balanced ranked set samples we need exactly n^2 units to select a sample of n units. A more reasonable and practical approach was proposed by Chen (2001) in parametric settings. They considered unbalanced RSS schemes for m sets of size n, drawn from the population, and each of them was ranked by judgment. Let $Y(i:n)j(k)$ denote the ith order statistic from the jth sample of size m in the kth cycle, then to get a larger sample the cycle can be repeated k times to get a sample of size mk :

$$\begin{array}{cccccc}
Cycle(1) & -- & Y_{(1:n)1(1)} & Y_{(2:n)2(1)} & \cdots & Y_{(m:n)m(1)} \\
Cycle(2) & -- & Y_{(1:n)1(2)} & Y_{(2:n)2(2)} & \cdots & Y_{(m:n)m(2)} \\
\vdots & & \vdots & \vdots & \vdots & \vdots \\
Cycle(k) & -- & Y_{(1:n)1(k)} & Y_{(2:n)2(k)} & \cdots & Y_{(m:n)m(k)}
\end{array}$$

An unbiased estimator for the population mean in this case is:

$$\bar{y}_{RSS}=\frac{1}{mk}\sum_{i=1}^{n}\sum_{k=1}^{K}Y_{(i:n)j(k)} \tag{1.1}$$

The efficiency of the RSS procedure depends on the ranking within the subsets or groups and the accurate arrangement is the goal of the RSS procedure, which in some cases may not be possible. In many cases, some helpful information is available prior to sampling, enabling sampling units to be reasonably ranked according to the variable of interest. This information can be obtained through visual inspection, prior knowledge about the sampling units from the results of previous sampling surveys, associate variables, and expert opinion, allowing observations to be ranked based on one or a combination of these source.This helping information can also be utilized to estimate non-response values in data. Many techniques, such as extreme-RSS and median-RSS, are proposed for obtainting estimates in case of non-response (Bouza et al., 2010).

High non-response is a very common problem in sample surveys today, statistically, to concerned about the increasing bias and disparity in estimating population quantities such as totals or means. Nonresponse also caused the loss in the accuracy of the survey estimates, mainly due to the lower sample size and secondly due to increased variation in the weights of the questionnaire.

Various methods have been proposed in the literature to adjust and compensate for non-response bias. Many of these methods utilize auxiliary or helping variables to estimate non-response units. Calibration is one of these techniques and was first proposed by Deville and Särndal (1992) to estimate population parameters. The calibration weighting has been widely used to adjust for non-response bias as well. The calibration estimator for the population mean is a weighted estimator that uses calibration weights that are as close as possible to the original sampling design weights while satisfying a set of constraints. These constraints are called calibration or benchmark constraints. For defining a classical calibration estimator for simple random sampling, let y_i be the value of ith observation of the study variable and $x_i = (x_{1i}, x_{2i}, \dots, x_{pi})$ for $q = 1,2, \dots P$ is the auxiliary vector associated with y_i and at least population totals of all P auxiliary variables are known before sampling and estimation to estimate the population total of study variable $Y = \sum_U y_i$. Let the total of jth variable be $T_{xq} = \sum_U x_{qi}$ and a vector of total(s) of p auxiliary variables is denoted by X , where $\sum_U$ is the sum of all $k \in U$. Also, the vector of Horvitz Thompson estimators of population total(s) for auxiliary variables is $\hat{t}_{x\pi} = \sum_s d_i x_i$ and $\sum_S$ is the sum on all $k \in s$.

The calibration estimator for estimating the population total of study variable y that is defined as

$$\hat{t}_{yc} = \sum_s w_i y_i \tag{1.2}$$

The weights w_i are named as calibration weights because these weights have a minimum distance from design weights and satisfy a calibration to benchmark constraints:

$$\sum_s w_i y_i = X = \sum_U x_i . \tag{1.3}$$

By using different distance functions, many linear and nonlinear calibration estimators can be obtained. The chi-square type distance functions generate calibration weights that are linear functions of the design weights and can be written as:

$$w_i = d_i(1 + d_i \, x'_i \lambda) \tag{1.4}$$

The resulting calibration estimator is approximately equal to the general linear regression estimator,

$$\hat{t}_{yc} = \hat{t}_{y\pi} + b'_{ws}(X - \hat{t}_{x\pi})$$

where $\hat{t}_{y\pi} = \sum_s d_k y_k$ is the Horvitz Thompson estimators for the population total of the study variable and $b_{ws} = (\sum_s d_i \, q_i x_i \, x'_i)^{-1}(\sum_s d_i \, q_i x'_i y_i)$.

The calibration weighting is widely applied in surveys for adjusting non-response and correcting errors other than sampling errors. These methods typically address the group of respondents as a sample of the second stage, where the components of the double-response group are associated with compensation for both sampling and non-response (Bouza, 2013). These weights, especially those for non-response observations are created with the help of additional information. Kott (2015) suggested that the calibration weights can be used to remove bias when the non-response units are a function of one or more variables in the survey. It was done by allowing the model variables in the weight control function to change the variables in the calibration equation. Another study was by Sinha et al. (2017) who used calibration weights to adjust non-response using auxiliary variables available at the estimation stage. A new method of calibration weighting was proposed under the Stratified simple random sampling design (SSRS). It was proved that the calibration capabilities under RSS were better than SSRS. Some of the researchers have also proved that the RSS calibration measures performed better than the SSRS and SRS (Koyuncu, 2018). Recently Singh et al. (2021) considered the calibration techniques estimation of population variance of the response variable under random non-response and proposed a logarithmic type estimator using information available on a highly positively correlated auxiliary variable. Calibration techniques have been applied to determine the optimum strata weight. The results of Empirical studies showed that both for real and simulated data, the variance of the response variable is more efficient under calibration techniques for random non-response. Recently, Mehreen et al. (2022) proposed an estimator for the estimation of population mean in the presence of non-response in study variable by using ranked set sampling procedure for asymmetrical distributions and showed that use of ranked set sampling reduces the effect of asymmetry in the characteristics under study. It was found that the proposed estimator was more efficient than the compared estimators in the case of two auxiliary variables.

In this study non-response in RSS is adjusted through a calibration approach. A Calibration estimator for the population mean in case of full and non-response cases is proposed. The expression for Mean square error is derived in both cases and simulation studies are performed to determine the efficiency of the proposed estimators.

2. NON-RESPONSE ADJUSTMENT IN RANKED SET SAMPLING USING CALIBRATION WEIGHTING

Let a vector of auxiliary variable $X_{(i:n)j} = \left\{1, X_{(i:n)j(1)} \ldots X_{(i:n)j(p)}\right\}$ is associated with study variable's value $Y_{(i:n)j}$ and assume that all values of auxiliary variables corresponding to the sampling units of the study variable are known prior to sampling and are correlated with study variable values. The proposed calibration estimator for population mean in this case (full response) is defined as:

$$\bar{y}_{CRSS} = \frac{\hat{y}_{TCRSS}}{mk} \text{ for } \hat{y}_{TCRSS} = \sum_j \sum_k w_{(i:n)j} y_{(i:n)j} \tag{2.1}$$

where $w_{(i:n)j}$ are obtained such that these weights have a minimum distance from sampling design weights $d_{(i:n)j}$ where $d_{(i:n)j} = \dfrac{1}{\pi_{(i:n)j}}$ and satisfy a calibration to benchmark constraints:

$$\sum_{(i:n)=1}^{n} w_{(i:n)j} x_{(i:n)j} = \sum_{(i:n)=1}^{N} X_{(i:n)j} \tag{2.2}$$

Let for the second case, we suppose that some of the values $Y_{(i:n)j}$, are missing and the order of missing values can be determined by using information of correlated auxiliary variable used in estimation and is known from any previous source:

$$\begin{array}{ccccc}
(Y_{(1:n)1}, X_{(1:n)1}) & \cdots & (Y_{(i:n)1} X_{(i:n)1}), & \cdots & (Y_{(n:n)1}, X_{(n:n)1}) \\
(Y_{(1:n)2}, X_{(1:n)2}) & \cdots & (Y_{(i:n)2}, X_{(i:n)2}) & \cdots & (Y_{(n:n)2}, X_{(n:n)2}) \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
(Y_{(1:n)j}, X_{(1:n)j}) & \cdots & (\square, X_{(i:n)j}) & \cdots & . \rightarrow\rightarrow Y_{(i:n)j} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
(Y_{(1:n)m}, X_{(1:n)m}) & \cdots & (Y_{(i:n)m}, X_{(i:n)m}) & \cdots & (Y_{(n:n)m}, X_{(n:n)m})
\end{array}$$

The proposed calibration approach is to divide the units of the auxiliary variables into two non-overlapping sets; one consists of the values that are paired with the response set of values of the study variable and the other for non-response set of values. The calibration weights for the non-response values of the study variable are obtained by satisfying a calibration to the sample totals of auxiliary variables.

Let $\sum_{(i:n)=1}^{n} x_{(i:n)}$ is the sample total of ordered auxiliary variable, r is the set of values $\sum_{(i:n)}^{r} x_{(i:n)}$ that are associated with the response set of study variable and $n-r$ is the set of non-response values $\sum_{(i:n)}^{n-r} x_{(i:n)}$

The non-response values can be estimated using the calibration weights that satisfy a calibration to the following benchmark constraints:

$$\sum_{(i:n)}^{n-r} w_{(i:n)nr} x_{(i:n)} = \sum_{(i:n)}^{n} x_{(i:n)j} \tag{2.3}$$

And non-response values can be estimated using

$$\hat{y}_{CRSS} = w_{(i:n)nr} \bar{y}_{RSS(r)} \tag{2.4}$$

where $\bar{y}_{RSS(r)}$ is the mean obtained from response values in ranked set sampling and the calibration weights $w_{(i:n)nr}$ are derived in such a way that they satisfy calibration to benchmark constraints (2.3) and have a minimum distance from sampling design weights and are obtained by differentiating Lagrange's equation $\angle = \frac{(w_{(i:n)nr} - d_{(i:n)})^2}{2d_{(i:n)}q_{(i:n)}} - \lambda_1 \left(\sum_{(i:n)}^{n-r} w_{(i:n)nr}x_{(i:n)} - \sum_{(i:n)}^{n} x_{(i:n)} \right)$ and putting it equal to zero. The $d_{(i:n)}$ is sampling design weight for i^{th} ordered observation selected in the ranked set sample. The resulting calibration weights are derived as:

$$w_{(i:n)nr} = d_{(i:n)} \left[1 + q_{(i:n)j} \lambda_1 x_{(\mathrm{i:n})} \right] \tag{2.5}$$

The value is obtained from calibration constraints (2.3). Using the value of λ_1 in equation (2.5), we get the resulting calibration weights for missing or non-response values.

$$w_{(i:n)nr} = d_{(i:n)j} \left[1 + q_{(i:n)j} \left(\frac{\sum_{(i:n)}^{n} x_{(i:n)} - \sum_{(i:n)}^{n-r} d_{(i:n)j} x_{(i:n)}}{\sum_{(i:n)}^{n-r} d_{(i:n)j} q_{(i:n)j} x_{.(i:n)}{}^{2}} \right) x_{(i:n)} \right] \tag{2.6}$$

Using calibration weights (2.6) in (2.4), we get the estimated values for missing observation of the study variable's values.

$$\hat{y}_{CRSS} = \left[\left\{ d_{(i:n)j} \left[1 + q_{(i:n)j} \left(\frac{\sum_{(i:n)}^{n} x_{(i:n)} - \sum_{(i:n)}^{n-r} d_{(i:n)j} x_{(i:n)}}{\sum_{(i:n)}^{n-r} d_{(i:n)j} q_{(i:n)j} x_{.(i:n)}{}^{2}} \right) x_{(i:n)} \right] \right\} \bar{y}_{RSS(r)} \right] \tag{2.7}$$

Using estimated values obtained in (2.7), the proposed calibration estimator in case of non-response in RSS is defined as:

$$\hat{y}_{TCRSS(nr)} = \sum_{(i:n)}^{n-r} w_{(i:n)nr} \hat{y}_{CRSS} + \sum_{(i:n)}^{r} w_{(i:n)r} y_{(i:n)} \tag{2.8}$$

where $w_{(i:n)r}$ are the calibration weights that satisfy calibration to benchmark constraints and have a minimum distance from the sampling design weights using Lagrange's equation.

$$\Gamma = \frac{\left(w_{(i:n)j} - d_{(i:n)} \right)^2}{2d_{(i:n)j} q_{(i:n)j}} - \lambda_2 \left(\sum_{(i:n)}^{n} w_{(i:n)j} x_{(i:n)j} - \sum_{(i:n)}^{N} X_{(i:n)j} \right) \tag{2.9}$$

$$w_{(i:n)r} = d_{(i:n)j} \left[1 + q_{(i:n)j} \lambda_2 x_{(i:n)j} \right] \tag{2.10}$$

The value is obtained from calibration constraints described in (2.2).

Utilizing value of λ_2 in equation (2.10), we get the resulting calibration weights to estimate the population total for response values

$$w_{(i:n)nr} = d_{(i:n)j}\left[1 + q_{(i:n)j}\left(\frac{\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j}x_{(i:n)j}}{\sum_{(i:n)}^{n-r} d_{ij}q_{ij}x_{(i:n)j}^{2}}\right)x_{(i:n)j}\right]$$

(2.11)

The weights (2.6) and (2.11) in (2.8), provide the resulting calibration estimator for estimating the population total of the study variable's values.

$$\hat{y}_{TCRSS(nr)} = \sum_{(i:n)}^{n-r}\left[d_{(i:n)j}\left\{1 + q_{(i:n)j}\left(\frac{\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j}x_{(i:n)j}}{\sum_{(i:n)}^{n-r} d_{ij}q_{ij}x_{(i:n)j}^{2}}\right)x_{(i:n)j}\right\}\hat{y}_{CRSS}\right]$$
$$+ \sum_{(i:n)}^{r}\left[d_{(i:n)j}\left\{1 + q_{(i:n)j}\left(\frac{\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j}x_{(i:n)j}}{\sum_{(i:n)}^{r} d_{ij}q_{ij}x_{(i:n)j}^{2}}\right)x_{(i:n)j}\right\}y_{(i:n)r}\right]$$

A simplified form of the estimator is, where

$$\hat{t}_{CRSS(nr)} = \frac{\hat{y}_{TCRSS(nr)}}{mk}$$

$$\hat{y}_{TCRSS(nr)} = \hat{t}_{yd(nr)} + b_{nr}\left(\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j}x_{(i:n)j}\right)$$
$$+\hat{t}_{yd(r)} + b_{r}\left(\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j}x_{(i:n)j}\right)$$

where

$$b_{nr} = \frac{\sum_{(i:n)}^{n-r} d_{(i:n)nr}q_{(i:n)nr}x_{(i:n)j}\,\hat{y}_{CRSS}}{\sum_{(i:n)}^{n-r} d_{(i:n)nr}q_{(i:n)nr}x_{(i:n)j}^{2}} \text{ and } b_{r} = \frac{\sum_{(i:n)}^{r} d_{(i:n)j}q_{(i:n)j}x_{(i:n)j}y_{(i:n)}}{\sum_{(i:n)}^{r} d_{(i:n)j}q_{(i:n)j}x_{(i:n)j}^{2}}$$

Also,

$$\hat{t}_{yd(nr)} = \sum_{(i:n)}^{n-r} d_{(i:n)nr}q_{(i:n)nr}\,\hat{y}_{CRSS} \text{ and } \hat{t}_{yd(r)} = \sum_{(i:n)}^{r} d_{(i:n)j}q_{(i:n)j}\,y_{(i:n)j} \qquad (2.12)$$

The estimator (2.12) uses calibration weights to adjust non-response values in RSS and is a weighted sum of calibration estimator in case of non-response that is the weighted calibration estimator of response and estimated non-response values.

3. COMPARISON OF THE PROPOSED ESTIMATORS

In ranked set sampling consider a finite population consisting of $N=\{Y_1,Y_2,Y_3,...,Y_N\}$ units, where $y_{(i:n)j}$ is *ith* value among the n ranked value in a set of j samples and with each unit of the study variable is associated with an auxiliary variable unit $x_{(i:n)j}$. The Population mean for the study variable is, $T_y=\dfrac{1}{N}\sum_{k=1}^{N}Y_k$ and the population mean for the auxiliary variable is $T_x=\dfrac{1}{N}\sum_{k}X_k$.

Also, the Horvitz-Thompson estimator for the population mean of the auxiliary variable in one cycle RSS can be defined as

$$\hat{t}_{dxRSS}=\frac{1}{m}\sum_{j}\sum_{i}d_{(i:n)j}x_{(i:n)j} \quad \text{where } i=1,...,n \quad j=1,...,m$$

The error associated with Horvitz Thompson estimator is $e_{dxRSS}=\hat{t}_{dxRSS}-T_x$.

Similarly, the error associated with the Horvitz-Thompson estimator for the population total of study variable in RSS $\hat{t}_{dyRSS}=\dfrac{1}{m}\sum_{j}\sum_{i}d_{(i:n)j}y_{(i:n)j}$ is $e_{dyRSS}=\hat{t}_{dyRSS}-T_y$.

Also, using the results $e_{RSS}=b_{RSS}-\beta_{RSS}$, $E\left(e_{dxRSS}^2\right)=\theta T_x^2C_{xRSS}^2$, $E\left(e_{dyRSS}^2\right)=\theta T_y^2C_{yRSS}^2$ and $E\left(e_{dxRSS}.e_{dyRSS}\right)=\theta T_xT_y\,C_{xRSS}\,C_{yRSS}\,\rho_{xy}$ where, $\rho_{xy}=\dfrac{S_{xyRSS}}{S_{xRSS}S_{yRSS}}$ and $\theta=\dfrac{N-n}{N.n}$.

Using the above results, the mean square errors of the proposed calibration estimator can be derived in case of full and non-response.

Case I:

To derive a Mean square error for the proposed estimator in case of full response we will follow the results derived by Deville and Särndal (1992); the calibration estimator is asymptotically equivalent to the regression estimator and bias related to the calibration estimator is

Hence MSE of the calibration estimator in ranked set sampling can be obtained in case of full response using the expression:

$$MSE(\bar{y}_{CRSS})=MSE(\bar{y}_{regR})+\left\{BiasO\left(n^{-1}\right)\right\}^2$$

After simplification, the mean square error of the calibration estimator in ranked set sampling when all sample values are known is obtained as:

$$MSE(\bar{y}_{CRSS}) = \theta T_y^2 C_{yRSS}^2 \left[1+\rho_{xy}^2\right] + \left\{BiasO\left(n^{-1}\right)\right\}^2$$

Case 2:

The mean square error of the proposed calibration estimator in case of non-response can be derived using the same results given in section 4.1.

The proposed calibration estimator in non-response case is,

$$\hat{t}_{CRSS(nr)} = \sum_{(i:n)}^{n-r} w_{(i:n)nr}\hat{y}_{CRSS} + \sum_{(i:n)}^{r} w_{(i:n)r} y_{(i:n)}$$

And can be written in the form:

$$\hat{t}_{CRSS(nr)} = \hat{t}_{yd(nr)} + b_{nr}\left(\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j} x_{(i:n)j}\right)$$
$$+ \hat{t}_{yd(r)} + b_r\left(\sum_{(i:n)}^{N} X_{(i:n)j} - \sum_{(i:n)}^{n} d_{(i:n)j} x_{(i:n)j}\right)$$

The Mean Square error of the estimator will be:

$$MSE\left(\hat{t}_{CRSS(nr)}\right) = E\left(\hat{t}_{CRSS(nr)} - T_y\right)^2$$

Using the results we have,

$$MSE\left(\hat{t}_{CRSS(nr)}\right) = \theta T_y^2 C^2_{(nr)yRSS}\left[1+\rho_{xy}^2\right] + \theta T_y^2 C_{(r)yRSS}^2\left[1+\rho^2{}_{xy}\right] + \left\{BiasO\left(n^{-1}\right)\right\}^2.$$

4. SIMULATION STUDY

To check the performance of the proposed estimators two simulation studies are carried out using artificially generated populations and real-life data; One for the full response and other for the non-response case

The proposed calibration imputation method for non-response estimation is compared with mean and median imputation in RSS. Also, the efficiency of the proposed calibration estimator $(\bar{y}_{CRSS})$ is compared with classical ranked set sample unbiased estimator of the population mean

$$\bar{y}_{RSS} = \frac{1}{mk}\sum_{i=1}^{n}\sum_{k=1}^{K} Y_{(i:n)j(k)}$$

and Double Median Ranked Set Sample estimator proposed by Samawi & Tawalbeh (2002) $\bar{y}_{DMRSS} = \frac{1}{mk}\sum_{i=1}^{n}\sum_{k=1}^{K} W_{\left(i\left(\frac{r+1}{2}\right)\right)(k)}$ for odd cases and $\bar{y}_{DMRSS} = \frac{1}{mk}\sum_{i=1}^{n}\sum_{k=1}^{K} W_{\left(i\left(\frac{r}{2}+1\right)th\right)(k)}$ for even cases where W_i are second stag median ranked set sample units.

- **Simulation 1**

In Simulation Study I, three populations are considered to evaluate the performance of the proposed estimator in case of full response as compared to previously existent estimators. For this purpose, two artificial populations and one real-life data was used. The the first population follows a normal distribution having large variance with parameters $N \sim (100{,}225)$. To assess the performance of the proposed estimators for highly skewed distributions, the second population follows a gamma distribution with parameters $U =$ Gamma (2, 2). Both populations consist of 10,000 units. From these populations, 100 ranked set samples were selected for each of the sample sizes: 5, 8, 10, and 25, using a one-cycle balanced ranked set sampling procedure (Sevinç et al., 2019).

For the first population, a correlated concomitant variable (X) was generated for the 10,000 units. This was done in such a way that ε_i follows a normal distribution, and the correlation coefficient between X and Y was fixed at 0.7. For the second population, the correlated variable (X_G) was generated using the relationship $X_G = U + 2V$, where U follows a Gamma (2, 2) distribution and V follows a Gamma(1, 2) distribution. This ensured that the correlation between U and X_G is $\rho = 0.623$.

The third population was real-life data taken from the online source (http://mercury.webster.edu/aleshunas/Data%20Sets/Supplemental%20Excel%20Data%20Sets.htmto) consisting of 2445 rows. The response variable "sales of items per day" follows a normal distribution (Kolmogorov Smirnov test's p-value = 0.535) with ($\mu = 3.5$, $\sigma = 2.112$) and the auxiliary variable "Price of items" is moderately correlated ($r = 0.650$) with the study variable. The performance of the estimators under full response case was evaluated for ranked set samples of sizes 5, 8, 10, and 25. Results for each sample size were averaged over 100 ranked set samples using one cycle Balanced Ranked Set Sampling (BRSS).

The performance of the proposed estimator is compared with two classical and commonly employed estimators for the population mean in the context of RSS. The estimators employed in the simulation study are outlined below:

i. The classical Mean estimator ($\bar{y}_{RSS}$)
ii. Double Median Ranked set sample estimator ($\bar{y}_{DMRSS}$).
iii. Calibration Estimator ($\bar{y}_{CRSS}$)

The population mean in ranked set sampling $T_{yRSS} = \frac{1}{N}\sum Y_{n(i:n)}$ is estimated using the proposed calibration estimator ($\hat{t}_{CRSS(nr)}$) when missing values are imputed using the following three methods

i. Mean Imputation
ii. Median Imputation
iii. Imputation through calibration approach

For comparing the efficiency of the estimator, the Bias and MSE were calculated and defined as:

$$Bias(\hat{t}) = E(\hat{t}) - \theta \text{ and } MSE(\hat{t}) = E(\hat{t} - \theta)^2$$

- **Simulation 2**

In the second simulation study, we examined the effectiveness of the estimators in case of non-response. The efficiency of the proposed calibration estimator in case of non-response is investigated by imputing the missing values with the mean, median, and proposed calibration imputation methods. To induce non-response in each population, random missing values were deliberately introduced, accounting for a non-response rate of 20-25% of the data.

5. RESULTS

From Table 1, it is evident that for artificially generated populations, both bias and mean square progressively decrease as the sample size of RSS increases. In the case of the Normal distribution with large variance, the Double Median Ranked set sample estimator ($\bar{y}_{DMRSS}$) outperforms the classical mean estimator, although it is not as efficient as the proposed calibration estimator.

Also, the efficiency of the Calibration estimator increases with sample size, and a substantial reduction in bias and mean square error can be gained. The relative efficiency of the calibration estimator increases by 13% as compared to the Classical mean estimator and approximately 12% when compared with the ($\bar{y}_{DMRSS}$) for a BRSS of size 25. In the case of Gamma (2, 2) distribution, the classical mean estimator outperforms the median estimator for an even number of observations in the sample that is the RSS of size $n = 8$ and 10. However, in this scenario, the proposed estimator consistently demonstrates the highest level of efficiency among the compared estimators. The mean square error of the Calibration estimator reduces to 0.001431 in the case of Gamma distribution with RSS of size 25 which is negligible as compared to other estimators. The same pattern can be observed in the case of real data. The optimum value of mean square error is obtained (0.03908) for the Calibration estimator under RSS for $n = 25$.

Table 2 shows the results of the simulation study in case of non-response. It can be observed that a substantial reduction in bias and mean square error is attained using imputation through calibration weighting. The estimator of population mean that uses the calibration imputation method performs better than the other estimators in terms of minimum bias and mean square Error. The bias and mean square error reduce gradually with increasing the sample size but the optimum value for mean square is achieved for the case when the calibration imputation method is used. The mean imputation method outperforms median imputation method for the generated normal population, however, the $\bar{y}_{DMRSS}$ performs better in the other cases. However, the calibration weighting imputation consistently demonstrates the highest level of efficiency among the compared imputation methods for non-response. The relative efficiency of the proposed method increases by 16.67% as compared to mean imputation and 24% as compared to median imputation for RSS of $n = 25$ in the case of normally distributed data with large variance. Also, the MSE reduces to 0.0021184 for Gamma (2,2) and 0.1007 for real data respectively when the calibration imputation method is used.

6. CONCLUSION

Two estimators for the population mean are presented within the framework of ranked set sampling. One is designed for situations of full response, while the other addresses cases of non-response by employing a proposed calibration imputation method for adjustment. The Mean Square Error (MSE) and Bias of each estimator are derived. Additionally, two simulation studies are conducted to evaluate the performance of these estimators in both full response and non-response scenarios.

The findings demonstrate that the proposed calibration estimator for full response outperforms the compared estimators in terms of efficiency. Moreover, imputation based on calibration weighting exhibits greater efficiency, with lower values of bias and mean square error compared to the alternative imputation methods. In conclusion, it can be affirmed that the calibration estimator in ranked set sampling proves to be more efficient than the compared estimators. Furthermore, imputation using calibration weighting for handling non-response or missing values in ranked set sampling leads to more accurate estimates when contrasted with mean or median imputation methods.

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Table 1
Results of Simulation Study for Generated and Real Life Populations in case of Full Response

Estimators	Sample Sizes	Generated Data								Real Data			
		Normal (100,225)				Gamma(2,2)				Normal(3.5, 4.11)			
		5	8	10	25	5	8	10	25	5	8	10	25
$\bar{y}_{RSS}$	Mean	95.2286	96.4053	96.866	98.203	1.2218	0.98610	1.01711	1.00822	2.9876	3.05796	3.3062	3.6681
	Bias	-4.4521	-3.4755	-2.0141	-1.6775	-0.2447	0.13971	0.12058	-0.0647	-0.8967	-0.6139	-0.4067	0.3982
	MSE	12.1088	7.0301	5.0850	2.5591	0.05990	0.01651	0.01454	0.00419	0.8099	0.3713	0.1941	0.0966
$\bar{y}_{DMRSS}$	Median	103.206	96.9504	96.971	102.68	0.88593	1.23549	0.96284	1.02245	3.02102	3.1714	3.2500	3.7734
	Bias	3.32577	-2.9304	-2.0093	1.6002	-0.1078	0.29764	-0.2309	0.00867	-0.7567	-0.4853	-0.3167	0.3032
	MSE	11.0607	6.58742	6.1076	2.2409	0.00989	0.04815	0.03156	0.00822	0.5467	0.2355	0.1640	0.1964
$\bar{y}_{CRSS}$	Mean	96.2670	97.8526	97.929	98.902	0.94361	0.99758	0.99767	0.98999	3.7016	3.6682	3.6510	3.4549
	Bias	-3.1137	-2.0279	-1.4508	1.2173	-0.0801	-0.0501	-0.0541	-0.0378	0.4284	0.3497	0.3133	-0.1976
	MSE	10.0593	4.0566	3.0041	1.9731	0.00951	0.00580	0.00583	0.00143	0.2497	0.1711	0.1547	0.03908

Table 2
Results of Simulation Study for Generated and Real Life Populations in case of Non-Response

Estimators	Sample Sizes	Generated Data								Real Data			
		Normal (100,225)				Gamma(2,2)				Normal(3.5, 4.11)			
		5	8	10	25	5	8	10	25	5	8	10	25
Mean	Mean	92.8876	93.4579	103.176	95.0681	0.94321	1.394973	0.96781	1.208222	2.7876	2.8779	3.3062	3.6981
	Bias	-7.0095	-3.42287	4.29537	-3.81267	-0.54331	0.41332	-0.22580	0.220759	-0.998	-0.9139	-0.4067	0.3982
	MSE	16.0542	9.40752	8.4502	4.53645	0.23961	0.20112	0.11359	0.10431	0.8993	0.7913	0.2941	0.1666
Median	Median	112.323	95.6834	104.958	95.0681	0.95933	0.963549	1.26846	0.97456	3.0210	3.0714	3.1500	3.6734
	Bias	13.4421	-3.5974	6.07797	-3.99126	-0.50755	-0.476432	0.29031	-0.200670	-0.8567	-0.5853	-0.4367	0.3432
	MSE	18.6919	10.2233	9.94172	6.3364	0.09989	0.098159	0.09663	0.08350	0.7467	0.7355	0.3640	0.1964
Calibration	Mean	95.3524	97.1310	97.4874	98.6831	0.96361	0.975871	0.99767	1.009999	3.8096	3.7682	3.6610	3.6549
	Bias	-4.5284	-1.7497	-1.9331	-1.2976	-0.83301	-0.50312	-0.54311	-0.047811	0.8284	0.5497	0.3933	0.2776
	MSE	10.4497	6.06171	4.1547	2.73908	0.09251	0.044802	0.02489	0.002118	0.6417	0.3271	0.2334	0.1007