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STEP STRESS PARTIALLY ACCELERATED LIFE TESTING PLAN FOR COMPETING RISK USING ADAPTIVE TYPE-I PROGRESSIVE HYBRID CENSORING

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ABSTRACT

This study deals with estimating information about failuretimes of items under step-stress partially accelerated life tests for competing risk based on adaptive type-I progressive hybrid censoring criteria. The life data of the units under test is assumed to follow the Weibull distribution. The point and interval maximum likelihood estimations are obtained for distribution parameters and tampering coefficient. The performance of the resulting estimators of the developed model parameters are evaluated and investigated by using a simulation algorithm.

KEYWORDS

Competing risk, Partially accelerated life testing, Weibull distribution, Adaptive progressive hybrid censoring, Simulation study.

1. INTRODUCTION

In the modern era, it is very hard to obtain information in regard to lifetime of items or systems with high reliability under usual operating conditions. In such problems, an experimental process called "*accelerated life testing*" (ALT) is conducted, where products are tested under higher stress than normal to find and induce their failure information. Commonly used stress patterns are step-stress and constant-stress (Nelson 1990). Thus, ALTs or partially accelerated life tests (PALTs) are conducted to minimise the lives of systems or items and to reduce the experimental time and the cost incurred in the experiment. Using step-stress PALT (SSPALT), a product or system is first subjected to normal (use) conditions for a pre-specified duration of time, and if it survives then it is put into service at accelerated condition until the termination time of the experiment.

Although PALT procedure can be conducted to shorten the test time of an experiment but it still takes a long time to wait for all the units to get failed. Therefore, censoring schemes has been an important tool to consider. (Commonly used censoring schemes are Type-I and Type-II). The most common in use are Type-I and Type-II censoring schemes (see, e.g., Balakrishnan and Ng (2006)). Suppose there are n units under particular experimental considerations. Under the traditional Type-I censoring plan, the specimens are tested up to a pre-fixed time point T_0. On the other hand, the Type-II censoring plan needs the process to continue until the pre-specified number of failures $m \le n$ are obtained. The mixture of these two censoring schemes is called as hybrid censoring scheme. Many of the researchers have used this censoring scheme as a tool for their research (see, e.g., Gupta and Kundu (1988), Childs et al. (2003), Kundu (2007) and N. Balakrishnan and Debasis Kundu (2013)). One of the major limitations of this censoring scheme is the lack of flexibility to remove some units from the experiment at any point other than the termination point. To overcome this drawback, a more general censoring scheme called progressive Type-I censoring scheme or progressively Type-I hybrid censoring scheme are introduced. In order to overcome this limitation and for the further efficiency of the estimate of the parameters, (Ng et al., 2009) introduced another censoring scheme called adaptive type-II censoring scheme. In this censoring scheme the experimenter prefix the effective sample size m under the given corresponding progressive censoring scheme, however at each failure time, the items progressively taken off from the experiment may change in number.

Recently, Lin and Huang (2012), presented an advanced hybrid censoring scheme called an adaptive type-I progressive hybrid censoring scheme (AT-I PHCS). Here, n identical specimens are placed under an experiment with the progressive censoring scheme $R_1,...,R_m$, $m \in [1,n]$; and the experiment is permanently stopped at a prefixed time T_0, where R_i's are whole numbers fixed in advance. At each failure time $(t_{1,m,n},\ t_{2,m,n}\ ,...)$, $(R_1, R_2\ ,...)$ of the remaining units are randomly taken off from the test, respectively. When the first failure $t_{1,m,n}$ occurs, randomly R_1 test items from the experiment are removed. Similarly, at the time when the second unit fails $x_{2,m,n}$, remove R_2 items from the remaining ones and so on. Suppose J denote the total number of units that failed before or up to time T. If possible suppose the m^{th} failure $t_{m,m,n}$ occurs before time T_0 (i.e. $t_{m,m,n} < T_0$), then the experiment will not be terminated but will go on to perceive failures without any further removals until time T_0. Finally, at time point T all the remaining units $R_J^* = n - J - \sum_{i=1}^{J} R_i$ are taken off from the test and the experiment automatically gets terminated. In this case, the progressive censoring design becomes $R_1, R_2, ..., R_m, R_{m+1}, ..., R_J$, where $R_{m+1} = R_{m+2} = ... = R_J = 0$. On the other hand, if $t_{m,m,n} > T_0$, the process will constitute a progressive censoring scheme as $R_1, R_2, ..., R_J$.

A lot of literature is available on SS-PALT analysis, see Goel (1971), DeGroot and Goel (1979), Bhattacharyya and Soejoeti (1989), Bai and Chung (1992), Abdel-Ghani (1998) and Abdel-Ghaly et al. (2002a, 2002b), Abdel-Ghani (2006), Ismail (2009), and Ismail (2012a), etc. Also, under hybrid censoring, Ismail (2012b) constructed SSPALT model for estimation purposes using Weibull failure data.

For an experimenter, it is equally important to judge the effect of failure cause apart from shortening the test time by using SSPALT along with different censoring schemes. It has become essential for an experimenter to recognise the difference between different failure causes in order to have exact information about failures. These failure causes are competing for the failure of the product. In hybrid censored life tests, Kundu and Gupta (2007) discussed competing risk model by assuming that the lifetimes of items under different causes of failure are independent exponential random variables. Competing risk has also been analysed by using SSPALT under progressively hybrid censoring. Competing risks model in SSPALT using progressively type-I hybrid censored data and Weibull distribution has also been discussed (Chunfang Zhang et al., 2016).

Based on the adaptive progressive hybrid censoring scheme using competing risk, few interesting studies have been made under ALT; for example, see Ashour and Nassar (2014, 2016). But there has been no previous study related to PALT in this aspect. **Hence,** this study will focus on adaptive progressively type-I hybrid censoring scheme using competing risk under step-stress PALT. The whole design under SSPALT using competing risk is discussed in section 2.

2. MODEL DESCRIPTION AND TEST METHOD

In modern life testing analysis, there may be more than one causes of an experimental unit to get failed. These "causes" are competing for the failure of an item. In this section, a design is framed to estimate the parameters and tempering coefficient in SSPALT under adaptive type-I progressive hybrid censoring scheme assuming that the failure causes are independent Weibull variables.

2.1 Basic Assumptions

1. Under SSPALT, the product is first tested at a normal stress level S_0 and at time τ the same is increased to S_1, $S_0 < S_1$.
2. At each stress level, there are p causes of a unit to get failed, denoted as $X_1, \ldots, X_p$.
3. Under normal stress level S_0, the hazard rate function (HRF) for each failure cause $X_k, k = 1, 2, \ldots, p$, is

$$h_k(x) = \beta_k \alpha_k^{-1} (x / \alpha_k)^{\beta_k - 1}, \quad x > 0; \ \beta_k > 0, \ \alpha_k > 0 \tag{1}$$

4. The HRF of X_k under SSPALT is given by tampered failure rate (TFR) model as

$$h_k^* = \begin{cases} h_{1k}(x) = h_k(x), & 0 < x \le \tau \\ h_{2k}(x) = \lambda_k h_k(x), & x > \tau \end{cases}$$

where, $\lambda_k = \lambda(>1)$, *for all* $k = 1, 2, \ldots p$, is a tempering coefficient. The corresponding

Survival function is given by

$$S_k^* = \begin{cases} S_{1k}(x) = \exp\left\{-(x / \alpha_k)^{\beta_k}\right\}, & 0 < x \le \tau \\ S_{2k}(x) = \exp\left\{-(\tau / \alpha_k)^{\beta_k} - \lambda_k \left[(x / \alpha_k)^{\beta_k} - (\tau / \alpha_k)^{\beta_k} \right]\right\}, & x > \tau. \end{cases} \tag{2}$$

5. In presence of independent failure causes, record the latent failure time as a joint random variable $(T,\ \gamma)$, where, $\gamma=\left(C_1,\ldots,C_p\right)$ and for $k=1,2,\ldots,p$

$$C_k=\begin{cases}1, & if\ T_i=X_k\\ 0, & if\ T_i\neq X_k.\end{cases}$$

2.2 The Testing under Type-I PHCS

Suppose n items are placed under test and let $t_1,t_2,\ldots,t_n$ be their corresponding lifetimes. Under SSPALT scheme the units are first subjected to normal stress level S_0 and then at time τ the stress is increased to S_1. Let m be the prefixed number of failure under both stress levels. The termination time T_0 along with removals $\left(R_1,R_2,\ldots,R_\tau,\ldots,R_m\right)$ is also fixed in advance. At the time of i^{th} failure $\left(t_{i:m:n},\gamma_i\right)$, R_i units are removed from the experiment and at the stress changing time τ, $R_\tau>0$ units would be withdrawn from the surviving ones and so on. Suppose j be the total number of failures that happen prior to time point T_0. In case the m^{th} failure $t_{m:m:n}$ occurs prior to time T_0 (i.e. $t_{m:m:n}<T_0$), the test will not stop, but will continue to perceive failures up to time T_0 without any further removals. Once the time T_0 is reached, all the remaining units $R_J^*=n-J-\sum_{i=1}^{J}R_i$ are removed and the test will terminate automatically. On the other hand, if the time T_0 is reached before the m^{th} failure (i.e. $t_{m:m:n}>T_0$) the test is terminated at the time T_0. The observed data in the SSPALT under adaptive type-I PHCS using competing risk is

Case I: when $t_{m:m:n}<T_0$

$$S_0=\left(t_{1:m:n},\gamma_1,R_1\right),\left(t_{2:m:n},\gamma_2,R_2\right),\ldots,\left(t_{n_u:m:n},\gamma_{n_u},R_{n_u}\right),\left(\tau,R_\tau\right),$$

$$S_1=\left(t_{n_u+1:m:n},\gamma_{n_u+1},R_{n_u+1}\right),\left(t_{n_u+2:m:n},\gamma_{n_u+2},R_{n_u+2}\right),\ldots,\left(t_{m-1:m:n},\gamma_{m-1},R_{m-1}\right),$$
$$\left(t_{m:m:n},\gamma_m,R_m\right),\left(t_{m:+1;m;n},\gamma_{m+1},0\right),\ldots,\left(t_{j:m:n},\gamma_j,0\right),\left(T_0,R_j^*\right)$$

Case II: $t_{m:m:n}=T_0$

$$S_0=\left(t_{1:m:n},\gamma_1,R_1\right),\left(t_{2:m:n},\gamma_2,R_2\right),\ldots,\left(t_{n_u:m:n},\gamma_{n_u},R_{n_u}\right),\left(\tau,R_\tau\right),$$

$$S_1=\left(t_{n_u+1:m:n},\gamma_{n_u+1},R_{n_u+1}\right),\left(t_{n_u+2:m:n},\gamma_{n_u+2},R_{n_u+2}\right),\ldots,\left(t_{m:m:n}=T_0,\gamma_m,R_m=R_j^*\right)$$

Case III: $t_{m:m:n}>T_0$

$$S_0=\left(t_{1:m:n},\gamma_1,R_1\right),\left(t_{2:m:n},\gamma_2,R_2\right),\ldots,\left(t_{n_u:m:n},\gamma_{n_u},R_{n_u}\right),\left(\tau,R_\tau\right),$$

$$S_1=\left(t_{n_u+1:m:n},\gamma_{n_u+1},R_{n_u+1}\right),\left(t_{n_u+2:m:n},\gamma_{n_u+2},R_{n_u+2}\right),\ldots,\left(t_{j:m:n},\gamma_j,R_j\right),\left(T_0,R_j^*\right)$$

where, n_u are the failure numbers at normal conditions.

The total failure number J and finally censored number R_J^* in SSPALT is

$$\begin{cases} J > m, \ R_J^* = n - J - \sum_{i=1}^{m} R_i, \ if \ \tau < t_{m:m:n} < T_0, \\ J = m, \ \ R_J^* = n - m - \sum_{i=1}^{m} R_i, \ if \ \ t_{m:m:n} = T_0, \\ J < m, \ R_J^* = n - J - \sum_{i=1}^{J} R_i, \ if \ \ t_{m:m:n} > T_0, \end{cases}$$

3. ESTIMATION PROCEDURE

The MLE is used here because it is very sound and gives the estimates of the parameters with good statistical properties. Here, in this section, we describe the point and interval estimation of the tempering coefficient and parameters of Weibull model based on adaptive type-I progressive hybrid censoring (APHC) using competing risk factor.

3.1. Point Estimation

This subsection discusses the procedure of obtaining the point ML estimates of parameters and tempering coefficient based on obtained data from APHC. The likelihood function under SSPALT using competing risk under given censoring scheme is obtained.

Let $t_1, t_2, ..., t_n$ be the n independently and identically distributed lifetimes of units following the Weibull distribution. The J completely observed (ordered) lifetimes are denoted by

$$t_{1:m:n} < ... < t_{n_u:m:n} \le \tau < t_{n_u+1:m:n} < ... \ < \ ... \ < \ t_{J:m:n} \qquad (*)$$

Based on the observed data and the assumptions discussed in section 2, the likelihood function under SSPALT using competing risk under the given censoring schemes is proportional to

$$L(\Theta / t) \propto \prod_{k=1}^{p} S_{1k}^{R_\tau}(\tau) \prod_{i=1}^{n_u} h_{1k}^{C_{ik}}(t_i) S_{1k}^{1+R_i}(t_i) \prod_{i=1}^{j} h_{2k}^{C_{ik}}(t_i) S_{2k}^{1+R_i}(t_i) S_{2k}^{R_j^*}(T) \qquad (4)$$

Substituting (1), (2) and (3) in the above function, we get

$$L(\Theta / t) \propto \prod_{k=1}^{p} \alpha_k^{-\beta_k r_k} \beta_k^{r_k} \lambda^{r_{2k}} \left(\prod_{i=1}^{j} t_i^{c_{ik}} \right)^{\beta_k - 1} \exp\left\{ -\alpha_k^{-\beta_k} \left(A_{1k} + A_{2k} \right) \right\} \qquad (5)$$

where,

$$r_k = r_{1k} + r_{2k} = \sum_{i=1}^{j} c_{ik}, \qquad r_{1k} = \sum_{i=1}^{n_u} c_{ik}, \ \ r_{2k} = \sum_{i=n_u+1}^{j} c_{ik}$$

$$A_{1k} = \sum_{i=1}^{n_u} t_i^{\beta_k} \left(1 + R_i\right) + \tau^{\beta_k} R_\tau$$

$$A_{2k} = \sum_{i=1}^{j}\left(1+R_i\right)\left[\tau^{\beta_k} + \lambda\left(t_i^{\beta_k} - \tau^{\beta_k}\right)\right] + R_j^{\bullet}\left[\tau^{\beta_k} + \lambda\left(T^{\beta_k} - \tau^{\beta_k}\right)\right].$$

Taking logarithm of the likelihood equation (5), we get

$$l = L\left(\Theta/t\right) = q\sum_{k=1}^{p}\left[r_k\left(In\beta_k - \beta_k In\alpha_k\right) + r_{2k}In\lambda + \left(\beta_k - 1\right)\sum_{i=1}^{j}c_{ik}Int_i - \alpha_k^{-\beta_k}\left(A_{1k} + A_{2k}\right)\right] \tag{6}$$

where, q is proportionality constant. We equate the partial derivatives of equation (6) to zero with respect the each parameter in the parameter set Θ, as

$$\frac{\partial l}{\partial\lambda} = \left(j - n_u\right)\lambda^{-1} - \sum_{k=1}^{p}\alpha_k^{-\beta_k}\left[\sum_{i=1}^{j}\left(1+R_i\right)\left(t_i^{\beta_k} - \tau^{\beta_k}\right) + R_j^{*}\left(T^{\beta_k} - \tau^{\beta_k}\right)\right] = 0 \tag{7}$$

$$\frac{\partial l}{\partial\beta_k} = \frac{r_k}{\beta_k} - r_k In\alpha_k + \sum_{i=1}^{j}c_{ik}Int_i + \alpha_k^{-\beta_k}\left[\left(A_{1k} + A_{2k}\right)In\alpha_k - \left(B_{1k} + B_{2k}\right)\right] = 0 \tag{8}$$

$$\frac{\partial l}{\partial\alpha_k} = -\frac{r_k}{\alpha_k} + \frac{A_{1k} + A_{2k}}{\alpha_k^{\beta_k+1}} = 0 \tag{9}$$

where,

$n_u = j - \sum_{k=1}^{p} r_{2k}$, r_{2k}, r_k, A_{1k} and A_{2k} are denoted in (6) for $k = 1, 2, ..., p$ and

$$B_{1k} \equiv \sum_{i=1}^{n_u}\left(1+R_i\right)t_i^{\beta_k}\, Int_i + R_j^{*}\tau^{\beta_k}In\tau.$$

$$B_{2k} \equiv \sum_{i=n_u+1}^{j}\left(1+R_i\right)C_{ik} + R_j^{*}C_{0k}$$

$$C_{ik} = \tau^{\beta_k}In\tau + \lambda\left(t_i^{\beta_k}Int_i - \tau^{\beta_k}In\tau\right)$$

From equation (8), the maximum likelihood estimator of α_k, for $k = 1, 2, \ldots, p$ can be obtained as

$$\hat{\alpha}_k = \left[\frac{\left(A_{1k} + A_{2k}\right)}{r_k}\right]^{\frac{1}{\beta_k}} \tag{10}$$

The popular Newton-Raphson iterative algorithm is employed to find the MLE's $\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_k$ and $\hat{\lambda}$ by substituting equation (10) into equation (8). When the values of $\hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_k$ are obtained, using (10), $\hat{\alpha}_k$ can be easily computed and hence $\Theta = \left(\hat{\lambda}, \hat{\alpha}_k, \hat{\beta}_k\right), k = 1, 2, ..., p$.

3.2 Interval Estimation

The approximate confidence interval of the model parameters based on the asymptotic distribution of the maximum likelihood estimators of the vector of unknown parameters $\Theta = \left(\hat{\lambda}, \hat{\beta}_1, \hat{\alpha}_1, \hat{\beta}_2, \hat{\alpha}_2, ..., \hat{\beta}_p, \hat{\alpha}_p\right)$. is derived. The variance-covariance matrix consisting of negative partial derivatives of equation (5) with respect to parameters is also constructed. For $k = 1, 2, \ldots, p$ we have;

$$I_{11} = -\frac{\partial^2 \ell}{\partial \lambda^2} = \frac{j - n_u}{\lambda^2}$$

$$I_{1(2k)} = -\frac{\partial^2 \ell}{\partial \lambda \, \partial \beta_k} = \alpha_k^{-\beta_k} \left[\sum_{i=n_u+1}^{j} \left(1 + R_i\right)\left(t_i^{\beta_k} Int_i - \tau^{\beta_k} In\tau\right) + R_j^* \left(T^{\beta_k} InT - \tau^{\beta_k} In\tau\right) \right]$$
$$- \alpha_k^{-\beta_k} \left[\sum_{i=n_u+1}^{j} \left(1 + R_i\right)\left(t_i^{\beta_k} - \tau^{\beta_k}\right) + R_j^* \left(T^{\beta_k} - \tau^{\beta_k}\right) \right],$$

$$I_{(2k)(2k)} = -\frac{\partial^2 \ell}{\partial \beta_k^2} = r_k \beta_k^{-2} + \alpha_k^{-\beta_k} \left[\sum_{i=1}^{n_u} \left(1 + R_i\right) t_i^{\beta_k} \left(In\, t_i - In\, \alpha_k\right)^2 + R_j^* \tau^{\beta_k} \left(In\tau - In\alpha_k\right)^2 \right]$$
$$+ \left[\sum_{i=n_u+1}^{j} \left(1 + R_i\right)\left(\left(1 - \lambda\right)\left(In\tau - In\alpha_k\right)^2 \tau^{\beta_k} + \lambda t_i^{\beta_k} \left(Int_i - In\alpha\ \right)^2\right) \right] \alpha_k^{-\beta_k}$$
$$+ \alpha_k^{-\beta_k} R_j^* \left[\left(1 - \lambda\right)\left(In\tau - In\alpha_k\right)^2 \tau^{\beta_k} + \lambda \left(InT - In\alpha_k\right)^2 T^{\beta_k} \right],$$

$$I_{(2k)(2k+1)} = -\frac{\partial^2 \ell}{\partial \beta_k \partial \alpha_k} = r_k \alpha_k^{-1} + \left[\left(A_{1k} + A_{2k}\right) In\alpha_k - \left(B_{1k} + B_{2k}\right)\right] \beta_k \alpha_k^{-(\beta_k + 1)},$$

$$I_{(2k+1)(2k+1)} = -\frac{\partial^2 \ell}{\partial \alpha_k^2} = -r_k \alpha_k^{-2} + \left(1 + \beta_k\right) \alpha_k^{-(\beta_k + 2)} \left(A_{1k} + A_{2k}\right),$$

Therefore we have the approximate 100(1-γ) % confidence intervals for λ, β and α, as

$$\lambda_k \pm Z_{\gamma/2} \sqrt{I_{11}}, \tag{13}$$

$$\hat{\beta}_k \pm Z_{\gamma/2} \sqrt{I_{(2k)(2k)}}, \tag{14}$$

$$\hat{\alpha}_k \pm Z_{\gamma/2} \sqrt{I_{(2k+1)(2k+1)}}, \tag{15}$$

Here $Z_{\gamma/2}$ is the $\left(\gamma / 2\right)$-th percentile of variate following $N\left(0,1\right)$.

4. SIMULATION STUDY

The simulation procedures to examine the performance of the ML estimators in respect of their confidence intervals and MSEs under different values of

$n, m, \tau,$ and T_0 are performed. Before starting the test, fix the testing conditions, i.e., normal stress level S_0, accelerated stress level S_1, sample size n, failure number m, removal numbers $(R_1, R_2, ..., R_m, R_\tau)$, competing risk number k, the stress changing time τ, the censored time T of type-1 APHCS, the values of parameters and tempering coefficient (acceleration factor). The simulated observed data $t = \left(t_1, t_2, ..., t_{n_u}, t_{n_u+1}, t_{n_u+2}, ..., t_m, t_{m+1}, ..., t_j, T_0\right)$ and approximated estimates in presence of competing risks under SSPALT with type-I APHCS are obtained. Based on the given assumption, the detailed procedures are given below:

1. For the failure time at the usual level of stress S_0, generate a progressive type-I censoring data $U = \left(u_{(1)}, u_{(2)}, ..., u_{(m)}\right)$ from $U(0,1)$ distribution with sample size $n - R_\tau$ and the removed units $(R_1, R_2, ..., R_m)$ by the simulation algorithm given by Balakrishnan and Sindu (1995).
2. Use the inverse CDF method to generate the type-k ordered samples $\left(t_{(1k)}, t_{(2k)}, ..., t_{(mk)}\right)$ under competing risk and attain the observed failure time $t_{i:m:n} = \min\left(t_{i1}, t_{i2}, ..., t_{1k}\right) < \tau$ and the indicator γ_i of failure causes for $1 \le i \le n_u$.
3. If $n_u = m$ or $\sum_{i=1}^{n_u} c_{ik} = 0,$ for any arbitrary cause of failure, restart the procedure from step 1 until $\sum_{i=1}^{n_u} c_{ij} \ge 1$ and $k \le n_u < m$, so that the failure time $t_1, t_2, ..., t_{n_u}, \tau$ can be obtained.
4. Now, for obtaining the failure time under accelerated stress level S_1, repeat the step in (1) to generate $U^\bullet = \left(u^\bullet_{(1)}, u^\bullet_{(2)}, ..., u^\bullet_{(m-n_u)}\right)$ with sample size $n - R_\tau - n_u - \sum_{l=1}^{n_u} R_l$ and withdrawn numbers $R_{n_u+1}, R_{n_2+2}, ..., R_m$.
5. Again, for given values of parameters we use inverse CDF method i.e. using $t = \frac{\alpha_k}{\lambda^{\beta_k^{-1}}}\left[-In\left(1-U^\bullet\right) - \left(\frac{\tau}{\alpha_k}\right)^{\beta_k}(1-\lambda)\right]^{\beta_k^{-1}}$ to generate the competing risk ordered sample $t_{\left(i(n_u+1)k\right)}, t_{\left(i(n_u+2)k\right)}, ..., t_{(jk)}$ and obtain the observed failure time $t_{i:m:n} = \min\left(t_{i1}, t_{i2}, ..., t_{ik}\right)$ and γ_i for $n_u + 1 \le i \le j$. The experiment is terminated at T_0 until J failures are obtained.

6. For arbitrary failure cause if $\sum_{i=n_u+1}^{j} c_{ik} = 0$, repeat the procedure in step (4) and make sure that $\sum_{i=n_u+1}^{j} c_{ik} \geq 1$ so as the failure times at accelerated stress level can be obtained.
7. The MLEs are obtained as $\Theta = \left(\hat{\lambda}, \hat{\beta}_1, \hat{\alpha}_1, \hat{\beta}_2, \hat{\alpha}_2, ..., \hat{\beta}_p, \hat{\alpha}_p\right)$. from equation (10) using the iterative technique. Also by using Equations (11), (12) and (13), we compute the asymptotic confidence intervals.
8. Replicate the whole procedure in steps 1-7 N times and find the mean estimates, MSEs and interval lengths (ILs) of $\hat{\Theta}$.
9. For different values of (n, m, τ, T_0) we specify $k = 2, (\lambda, \beta_1, \alpha_1, \beta_2, \alpha_2)$ under following five adaptive progressive hybrid censoring plans/schemes in SSPALT:
   a) $R_\tau = 0, R_1 = R_2 = ...R_{m-1} = 0, R_m = n - m,$
   b) $R_\tau = 0, R_1 = n - m;\ R_2 = ... = R_m = 0,$
   c) $R_\tau = 1, R_1 = R_2 = ... = R_{m-1} = 0, R_m = n - m - 1,$
   d) $R_\tau = 0, R_1 = n - m - 1, R_2 = ... = R_m = 0,$
   e) $R_\tau = n - m - 1,\ R_1 = R_2 = ... = R_m = 0,$

Table 1
Mean numbers of MLEs along with their MSEs and ILs when Setting $(\lambda, \beta_1, \alpha_1, \beta_2, \alpha_2) = (1.5, 2, 4, 4, 3)$, $(n, m, \tau, T_0) = (25, 5, 1.8, 2.2)$ & $(n, m, \tau, T_0) = (40, 10, 1.8, 2.2)$.

Values	(n, m, τ, T_0)	(25, 5, 1.8, 2.2)					(40, 10, 1.8, 2.2)				
	Scheme	(a)	(b)	(c)	(d)	(e)	(a)	(b)	(c)	(d)	(e)
λ	MLEs	1.942	2.112	1.812	2.132	1.784	1.835	1.652	1.675	1.675	1.672
	MSEs	0.846	1.519	0.671	1.639	0.092	0.792	0.679	0.612	0.615	0.075
	ILs	3.213	4.193	2.126	4.117	2.119	3.098	4.118	2.039	3.908	1.798
β_1	MLEs	2.731	2.674	2.998	2.677	2.563	2.367	2.354	2.654	2.315	2.490
	MSEs	0.671	0.697	0.903	0.691	0.467	0.645	0.602	0.719	0.587	0.361
	ILs	2.989	2.896	2.975	4.767	2.961	2.664	2.612	3.109	5.234	2.478
α_1	MLEs	3.127	3.263	3.212	3.133	3.953	3.564	3.342	3.712	3.198	4.515
	MSEs	0.891	0.267	0.511	0.897	0.192	0.673	0.219	0.510	0.612	0.169
	ILs	4.886	4.014	4.322	4.665	3.551	4.234	4.167	4.661	4.167	2.781
β_2	MLEs	3.244	3.415	3.122	3.419	3.501	3.812	3.776	4.017	3.998	3.798
	MSEs	0.829	0.899	0.913	0.998	0.752	0.776	0.791	0.817	0.917	0.681
	ILs	5.087	4.789	4.344	5.122	4.251	5.008	3.910	3.761	4.110	3.885
α_2	MLEs	3.011	2.567	2.051	2.452	3.150	3.361	3.297	3.122	3.197	3.151
	MSEs	0.139	0.409	0.500	0.699	0.172	0.322	0.211	0.202	0.201	0.118
	ILs	4.532	3.912	4.766	4.988	3.976	4.010	3.889	4.673	4.329	2.212

Table 2
Mean numbers of MLEs along with their MSEs and ILs when Setting
$(\lambda, \beta_1, \alpha_1, \beta_2, \alpha_2) = (1.5, 2, 4, 4, 3)$**,** $(n, m, \tau, T_0) = (70, 15, 1.8, 2.6)$
& $(n, m, \tau, T_0) = (100, 25, 2, 3)$

Values	(n,m,τ,T)	(70, 15, 1.8, 2.6)					(100, 25, 2.0, 3.0)				
	Scheme	(a)	(b)	(c)	(d)	(e)	(a)	(b)	(c)	(d)	(e)
λ	MLEs	1.661	1.355	1.786	1.746	1.567	1.617	1.417	1.592	1.677	1.553
	MSEs	0.446	0.547	0.762	0.766	0.113	0.398	0.488	0.600	0.667	0.108
	ILs	1.778	1.875	2.063	2.199	1.347	1.512	1.677	1.791	1.765	1.114
β_1	MLEs	2.454	2.398	2.437	2.511	2.167	2.313	2.301	2.308	1.799	2.144
	MSEs	0.519	0.476	0.461	0.600	0.300	0.277	0.155	0.267	0.178	0.645
	ILs	1.578	1.767	1.796	2.012	1.387	1.513	1.600	1.189	1.922	1.298
α_1	MLEs	4.410	4.519	4.378	4.614	4.286	4.409	4.336	4.257	4.455	4.133
	MSEs	0.797	0.695	0.601	0.886	0.370	0.674	0.518	0.549	0.675	0.211
	ILs	1.956	2.099	2.124	2.675	1.456	1.199	1.918	2.008	2.304	1.371
β_2	MLEs	4.447	4.577	4.378	4.483	3.876	4.499	4.593	4.310	4.334	3.905
	MSEs	0.890	0.900	0.717	0.756	0.409	0.808	0.688	0.877	0.706	0.365
	ILs	1.809	2.398	2..002	1.812	1.698	1.979	2.213	2.000	1.745	1.566
α_2	MLEs	3.399	3.334	3.564	2.671	3.307	3.318	3.311	3.437	2.709	3.257
	MSEs	0.689	0.699	0.867	0.555	0.437	0.657	0.516	0.676	0.456	0.275
	ILs	2.100	1.922	2.385	1.707	1.517	1.954	2.011	2.310	1.600	1.490

On the basis of MSE and interval lengths, the Table 1 and Table 2 provide the results of MLEs and perform better at censoring plan (e), when so many items are not removed at the stress changing time τ. In Table 1, the MSEs and ILs get smaller as we increase the ratio (m/n). Also, the more we test the items under accelerated conditions; the MLEs are best fit for the model.

6. CONCLUSION

The study provides the processes and simulated procedure for estimating failure time information under SSPALT for competing risk based on adaptive type-I hybrid censoring. The specimens' lifetimes are assumed to follow a Weibull distribution. The ML estimates are impossible to be produced in closed form and hence the Newton-Raphson technique as an alternative method is proposed to obtain them. Based on the asymptotic distribution of ML estimators, we construct and investigate the approximate confidence interval length of the parameters and tempering coefficient. Also, the performances of the resulting estimators are examined using MSEs by Monte-Carlo simulation procedure. From the results, it is found that the estimates are quite competent, especially in the case of sufficiently large sample size. In the end, as a future work, the same can be considered on adaptive progressive type-II hybrid censoring plan. Also, a Bayesian inference can be an interesting future work to do the same.

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