Pak. J. Statist.
2016 Vol. 32(6), 475-496

EFFICIENT CLASSES OF RATIO-TYPE ESTIMATORS OF POPULATION MEAN UNDER STRATIFIED MEDIAN RANKED SET SAMPLING

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ABSTRACT

In this paper, we propose two efficient classes of ratio-type estimators for estimating the finite population mean $(\bar{Y})$ under stratified median ranked set sampling (S_tMRSS) using the known auxiliary information. The biases and mean squared errors $(MSEs)$ of the proposed classes of ratio-type estimators are derived upto first order of approximation. The proposed estimators are compared with some competitor estimators. It is demonstrated through simulation study that the proposed ratio-type estimators based on S_tMRSS are more efficient than the corresponding estimators in stratified ranked set sampling (S_tRSS) given by Mandowara and Mehta [13].

KEYWORDS

Stratified ranked set sampling, Mean squared error, Ratio-type estimators, Stratified median ranked set sampling.

2000 Mathematics Subject Clasification: 62D05.

1. INTRODUCTION

Ranked Set Sampling (RSS) technique was first introduced by Mclntyre [14] and S_tRSS was suggested by Samawi and Muttlak [16] to obtain more efficient estimator for population mean. They also proposed an estimator of population ratio in *RSS* and showed that it has less variance as compared to ratio estimator in simple random sampling *(SRS)*. Takahasi and Wakimoto [20] showed that the sample mean under *RSS* is an unbiased estimator of the population mean and more precise than the sample mean estimator under *SRS*. Using S_tRSS, the performances of the combined and separate ratio estimates was obtained by Samawi and Siam [17]. Mandowara and Mehta [13] have adopted the Kadilar and Cingi [5] estimators in S_tRSS and obtained more efficient ratio-type estimators. Al-Saleh and Al-Kaddiri [2] have introduced the concept of double ranked set sampling ($DRSS$) and showed that $DRSS$ estimator is more efficient than the usual RSS estimator in estimating the finite population mean. As a modification of the RSS, Muttlak [17] has suggested the median ranked set sampling ($MRSS$) method for estimating the population mean. Jemain and Al-Omari [4] have suggested multistage median ranked set sampling for estimating the population mean. Al-Omari [1] has introduced modified ratio estimators in $MRSS$. Koyuncu [12] has proposed ratio and exponential type estimators in $MRSS$. Khan and Shabbir [7,8] proposed classes of Hartely-Ross type unbiased estimators in RSS and S_tRSS. Khan et al. [11] proposed unbiased ratio estimators of the finite population mean in S_tRSS. Khan and Shabbir [9,10] also proposed efficient classes of estimators in RSS.

In this paper, we use the idea of S_tMRSS in estimating the finite population mean and comparison is made with Mandowara and Mehta [13] estimators.

2. STRATIFIED RANKED SET SAMPLING

In S_tRSS, for the hth stratum, first choose m_h independent random samples each of size m_h ($h=1,2,...,L$). Ranked the observations in each sample and use RSS procedure to get L independent ranked set samples each of size m_h, to get $m_1+m_2+...+m_L=m$ observations. This completes one cycle of S_tRSS. The whole process is repeated r times to get the desired sample size $n=mr$.

To obtain Bias and MSE of the estimators, we define:

$$\bar{y}_{[S_tRSS]}=\bar{Y}(1+\varepsilon_0),\quad \bar{x}_{(S_tRSS)}=\bar{X}(1+\varepsilon_1)$$

such that $E(\varepsilon_i)=0$, $(i=0,1)$, and

$$E(\varepsilon_0^2)=\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{yh}^2-\frac{1}{m_h}\sum_{i=1}^{m_h}W_{yh[i:m_h]}^2\right),$$

$$E(\varepsilon_1^2)=\sum_{h=1}^{L}\frac{W_h^2}{m_h r}\left(C_{x_h}^2-\frac{1}{m_h}\sum_{i=1}^{m_h}W_{xh(i:m_h)}^2\right),$$

$$E(\varepsilon_0\varepsilon_1)=\sum_{h=1}^{L}\frac{W_h^2}{m_h r}(\rho_{yx_h}C_{y_h}C_{x_h}-\sum_{i=1}^{m_h}W_{yh[i:m_h]}W_{xh(i:m_h)},$$

where

$$W_{y_h[i:m_h]}=\frac{\tau_{y_h[i:m_h]}}{\bar{Y}},W_{x_h(i:m_h)}=\frac{\tau_{x_h(i:m_h)}}{\bar{X}},\ \tau_{x_h(i:m_h)}=\left(\mu_{x_h(i:m_h)}-\bar{X}_h\right)$$

and

$$\tau_{y_h[i:m_h]}=\left(\mu_{y_h([i:m_h]}-\bar{Y}_h\right).$$

Here $C_{y_h} = \frac{\sigma_{y_h}}{\bar{Y}}$ and $C_{x_h} = \frac{\sigma_{x_h}}{\bar{X}}$, $\mu_{y_h[i:m_h]} = E\left[y_{h[i:m_h]} \right]$ and $\mu_{x_h(i:m_h)} = E\left[x_{h(i:m_h)} \right]$; $\bar{Y}$ and $\bar{X}$ are the population means; $\bar{Y}_h$ and $\bar{X}_h$ are the population means of the h^{th} stratum of the variables Y and X respectively; ρ_{yx_h} is the population correlation coefficient between their respective subscripts in the h^{th} stratum.

Using S_tRSS, the combined ratio estimator of population mean $(\bar{Y})$ given by Samawi and Siam [17], is defined as

$$\bar{y}_{R(S_tRSS)1} = \bar{y}_{[S_tRSS]} \left(\frac{\bar{X}}{\bar{x}_{(S_tRSS)}} \right), \tag{1}$$

where $\bar{y}_{[S_tRSS]} = \sum_{h=1}^{L} W_h \bar{y}_{h[RSS]}$ and $\bar{x}_{(S_tRSS)} = \sum_{h=1}^{L} W_h \bar{x}_{h(RSS)}$ are the unbiased estimators of population means $\bar{Y}$ and $\bar{X}$ respectively; $W_h = \frac{N_h}{N}$ is the known stratum weight, N_h is the h^{th} stratum size, N is the total population size and L is the total number of strata $(h = 1, 2, ..., L)$.

Following Sisodia and Dwivedi [19], Mandowara and Mehta (2014) have suggested a modified ratio-type estimator for population mean $(\bar{Y})$ using S_tRSS, when population coefficient of variation of the auxiliary variable for the h^{th} stratum $\left(C_{x_h} \right)$ is known as

$$\bar{y}_{R(S_tRSS)2} = \bar{y}_{[S_tRSS]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h + C_{x_h} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(RSS)} + C_{x_h} \right)}. \tag{2}$$

Following Kadilar and Cingi [6], Mandowara and Mehta (2014) have proposed another ratio-type estimator for $\bar{Y}$ using stratified ranked set sampling as follows:

$$\bar{y}_{R(S_tRSS)3} = \bar{y}_{[S_tRSS]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h + \beta_{2(x_h)} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(RSS)} + \beta_{2(x_h)} \right)}. \tag{3}$$

Based on Upadhyaya and Singh [21], Mandowara and Mehta [13] have proposed two more ratio-type estimators, using both coefficient of variation and coefficient of kurtosis of the auxiliary variable in S_tRSS, are given by

$$\bar{y}_{R(S_tRSS)4} = \bar{y}_{[S_tRSS]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2(x_h)} + C_{x_h} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(RSS)} \beta_{2(xh)} + C_{x_h} \right)}, \tag{4}$$

and

$$\bar{y}_{R(S_tRSS)5} = \bar{y}_{[S_tRSS]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h C_{x_h} + \beta_{2(x_h)} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(RSS)} C_{x_h} + \beta_{2(x_h)} \right)}. \tag{5}$$

The Biases of $\bar{y}_{R(S_tRSS)1}$, $\bar{y}_{R(S_tRSS)2}$, $\bar{y}_{R(S_tRSS)3}$, $\bar{y}_{R(S_tRSS)4}$ and $\bar{y}_{R(S_tRSS)5}$, upto the first order of approximation are respectively, given by

$$Bias(\bar{y}_{R(S_tRSS)p}) \cong \bar{Y} \left[\begin{array}{l} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\lambda_p^2 C_{x_h}^2 - \lambda_p \rho_{yxh} C_{xh} C_{yh} \right) \\ -\sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \left(\lambda_p^2 \sum_{i=1}^{m_h} W_{x_h(i:m_h)}^2 - \lambda_p \sum_{i=1}^{m_h} W_{x_h(i:m_h)} W_{y_h[i:m_h]} \right) \end{array} \right], \tag{6}$$

where $p = 1, 2, \ldots, 5$.

and $\lambda_1 = 1$, $\lambda_2 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \left(\bar{X}_h + C_{x_h} \right)}$, $\lambda_3 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \left(\bar{X}_h + \beta_{2(x_h)} \right)}$,

$\lambda_4 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h \beta_{2(x_h)}}{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2(x_h)} + C_{x_h} \right)}$ and $\lambda_5 = \frac{\sum_{h=1}^{L} W_h \bar{X}_h C_{x_h}}{\sum_{h=1}^{L} W_h \left(\bar{X}_h C_{x_h} + \beta_{2(x_h)} \right)}$.

The MSE s of $\bar{y}_{R(S_tRSS)1}$, $\bar{y}_{R(S_tRSS)2}$, $\bar{y}_{R(S_tRSS)3}$, $\bar{y}_{R(S_tRSS)4}$ and $\bar{y}_{R(S_tRSS)5}$, upto the first order of approximation are respectively, given by

$$MSE(\bar{y}_{R(S_tRSS)p}) \cong \bar{Y}^2 \left[\begin{array}{l} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(C_{y_h}^2 + \lambda_p^2 C_{x_h}^2 - 2\lambda_p \rho_{yx_h} C_{x_h} C_{y_h} \right) \\ -\sum_{h=1}^{L} \frac{W_h^2}{m_h^2 r} \sum_{i=1}^{m_h} \left(W_{y_h[i:m_h]} - \lambda_p W_{x_h(i:m_h)} \right)^2 \end{array} \right] \tag{7}$$

where $p =, 2, \ldots, 5$.

3. MEDIAN RANKED SET SAMPLING (*MRSS*)

In $MRSS$ procedure, select m random samples each of size m from the population and rank the units within each sample with respect to a variable of interest. If the sample size m is odd, from each sample select for measurement the $((m+1)/2)$th smallest rank (i.e. the median of the sample). If the sample size is even, select for measurement from the first $m/2$ samples the $(m/2)$ th smallest rank and from the second $m/2$ samples the $((m+2)/2)$ th smallest rank. The cycle is repeated r times to get mr units. These mr units form the $MRSS$ data.

4. STRATIFIED MEDIAN RANKED SET SAMPLING

Ibrahim et al. [3] suggested stratified median ranked set sampling for estimating the population mean. To estimate the finite population mean $(\bar{Y})$ using S_tMRSS, the procedure can be summarized as follows:

Step 1: Select m_h^2 bivariate sample units randomly from the h^{th} stratum of population.

Step 2: Arrange these selected units randomly into m_h sets, each of size m_h.

Step 3: The procedure of $MRSS$ is then applied, to obtain L independent $MRSS$ samples each of size m_h, to get $m_1+m_2+...+m_L=m$ observations. Here ranking is done with respect to the auxiliary variable X.

Step 4: Repeat the above steps r times to get the desired sample size $n=mr$.

We use the following notations for the S_tMRSS when ranking is done with respect to the auxiliary variable X. For odd and even sample sizes the units measured using S_tMRSS are denoted by S_tMRSSO and S_tMRSSE, respectively. For the j^{th} cycle and the h^{th} stratum, the S_tMRSSO is denoted by $\left(Y_{1\left[\frac{m_h+1}{2}\right]j}, X_{1\left(\frac{m_h+1}{2}\right)j}\right), \left(Y_{2\left[\frac{m_h+1}{2}\right]j}, X_{2\left(\frac{m_h+1}{2}\right)j}\right), ..., \left(Y_{m_h\left[\frac{m_h+1}{2}\right]j}, X_{m_h\left(\frac{m_h+1}{2}\right)j}\right)$, $(j=1,2,...,r)$ and $h=1,2,...,L$. Let $\bar{y}_{h[MRSSO]}=\frac{1}{m_h}\sum_{i=1}^{m_h}Y_{i\left[\frac{m_h+1}{2}\right]}$ and $\bar{x}_{h(MRSSO)}=\frac{1}{m_h}\sum_{i=1}^{m_h}X_{i\left(\frac{m_h+1}{2}\right)}$ be the sample means of Y and X in h^{th} stratum respectively. For even sample size, the S_tMRSSE is denoted by

$$\left(Y_{1\left[\frac{m_h}{2}\right]j}, X_{1\left(\frac{m_h}{2}\right)j}\right), \left(Y_{2\left[\frac{m_h}{2}\right]j}, X_{2\left(\frac{m_h}{2}\right)j}\right), ..., \left(Y_{\frac{m_h}{2}\left[\frac{m_h}{2}\right]j}, X_{\frac{m_h}{2}\left(\frac{m_h}{2}\right)j}\right),$$

$$\left(Y_{\frac{m_h+2}{2}\left[\frac{m_h+2}{2}\right]j}, X_{\frac{m_h+2}{2}\left(\frac{m_h+2}{2}\right)j}\right), \left(Y_{\frac{m_h+4}{2}\left[\frac{m_h+2}{2}\right]j}, X_{\frac{m_h+4}{2}\left(\frac{m_h+2}{2}\right)j}\right), ..., \left(Y_{m_h\left[\frac{m_h}{2}\right]j}, X_{m_h\left(\frac{m_h}{2}\right)j}\right)$$

Let

$$\bar{y}_{h[MRSSE]} = \frac{1}{m_h}\left(\sum_{i=1}^{\frac{m_h}{2}} Y_{i\left[\frac{m_h}{2}\right]} + \sum_{i=\frac{m_h+2}{2}}^{m_h} Y_{i\left[\frac{m_h+2}{2}\right]}\right)$$

and $$\bar{x}_{h(MRSSE)} = \frac{1}{m_h}\left(\sum_{i=1}^{\frac{m_h}{2}} X_{i\left(\frac{m_h}{2}\right)} + \sum_{i=\frac{m_h+2}{2}}^{m_h} X_{i\left(\frac{m_h+2}{2}\right)}\right)$$

be the sample means in the h^{th} stratum.

To find Bias and MSE, we define:

$$\xi_{0(k)} = \frac{\bar{y}_{[S_t MRSSk]} - \bar{Y}}{\bar{Y}} \quad \text{and} \quad \xi_{1(k)} = \frac{\bar{x}_{(S_t MRSSk)} - \bar{X}}{\bar{X}},$$

such that $E(\xi_{i(k)}) = 0$, $(i = 0,1)$, where $k = (O, E)$ denote the sample size odd and even respectively. If sample size is odd, we can write:

$$E\left(\xi_{0(O)}^2\right) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2}\right),\quad E\left(\xi_{1(O)}^2\right) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2}\right),$$

$$E\left(\xi_{0(O)}\xi_{1(O)}\right) = \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right).$$

If sample size is even, we can write:

$$E\left(\xi_{0(E)}^2\right) = \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right),$$

$$E\left(\xi_{1(E)}^2\right) = \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{xh\left(\frac{mh+2}{2}\right)}}{\bar{X}_h^2}\right),$$

$$E\left(\xi_{0(E)}\xi_{1(E)}\right) = \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right).$$

5. FIRST PROPOSED CLASS OF ESTIMATORS

We propose the following class of estimators in S_tMRSS, given by

$$\bar{y}_{R(S_tMRSSk)p} = \bar{y}_{[S_tMRSSk]} \frac{\sum_{h=1}^{L} W_h \left(a_h \bar{X}_h + b_h\right)}{\sum_{h=1}^{L} W_h \left(a_h \bar{x}_{h(MRSS)} + b_h\right)}, \tag{8}$$

where a_h and b_h are known population parameters, which can be coefficient of variation, coefficient of skewness, coefficient of kurtosis and coefficient of quartiles of the auxiliary variable. Also, $k = (O, E)$ denote the sample size odd and even respectively. In terms of $\xi' s$, we have

$$\bar{y}_{R(S_tMRSSk)} = \bar{Y}(1+\xi_0)(1+\lambda\xi_1)^{-1},$$

where $\lambda = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h a_h}{\sum_{h=1}^{L} W_h \left(a_h \bar{X}_h + b_h\right)}$,

$$(\bar{y}_{R(S_tMRSSk)} - \bar{Y}) \cong \bar{Y}\left(\xi_0 - \lambda\xi_1 + \lambda^2\xi_1^2 - \lambda\xi_0\xi_1 + \ldots\right). \tag{9}$$

Taking expectations, we get biases of $\bar{y}_{R(S_tMRSSk)p}$, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_tMRSSO)p}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\lambda^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \lambda \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h} \right), \tag{10}$$

$$Bias\left(\bar{y}_{(S_tMRSSE)p}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\lambda^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) - \lambda \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right) \right]. \tag{11}$$

Squaring Equation (9) and then taking expectation, the $MSEs$ of $\bar{y}_{R(S_tMRSSk)p}$, for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)p}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \lambda^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\lambda \frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right], \tag{12}$$

$$MSE\left(\bar{y}_{R(S_t MRSSE)p}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2} \right) + \lambda^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) \right]$$
$$-\lambda \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right). \tag{13}$$

Note:

i) If $a_h = 1$ and $b_h = 0$, then from Equation (8), we get

$$\bar{y}_{R(S_t MRSSk)1} = \bar{y}_{[S_t MRSSk]} \left(\frac{\bar{X}}{\bar{x}_{(S_t MRSSk)}} \right), \tag{14}$$

The biases for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)1}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h} \right), \tag{15}$$

$$Bias\left(\bar{y}_{(S_t MRSSE)1}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) - \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right) \right]. \tag{16}$$

The $MSEs$ of $\bar{y}_{R(S_t MRSSk)1}$ for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)1}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right], \quad (17)$$

$$MSE\left(\bar{y}_{R(S_t MRSSE)1}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right) + \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right)\right]$$

$$-\bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right). \quad (18)$$

ii) If $a_h = 1$ and $b_h = C_{xh}$, then from Equation (8), we get

$$\bar{y}_{R(S_t MRSSk)2} = \bar{y}_{[S_t MRSSk]} \frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + C_{x_h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(MRSS)} + C_{xh}\right)}, \quad (19)$$

where C_{xh} is the population coefficient of variation of the auxiliary variable for the h^{th} stratum.

The biases of $\bar{y}_{R(S_t MRSSk)2}$ for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)2}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\lambda_1^{\,2} \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \lambda_1 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h}\right), \quad (20)$$

$$Bias\left(\bar{y}_{(S_t MRSSE)2}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\lambda_1^{\,2}\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right) - \lambda_1\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right)\right].$$

(21)

The $MSEs$ of $\bar{y}_{R(S_t MRSSk)2}$ for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)2}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \lambda_1{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\lambda_1 \frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right], \tag{22}$$

$$MSE\left(\bar{y}_{R(S_t MRSSE)2}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right) + \lambda_1{}^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right)\right]$$

$$-\lambda_1 \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right). \tag{23}$$

iii) If $a_h = 1$ and $b_h = \beta_{2(x_h)}$, then from Equation (8), we get

$$\bar{y}_{R(S_t MRSSk)3} = \bar{y}_{[S_t MRSSk]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h + \beta_{2(x_h)}\right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(MRSS)} + \beta_{2(x_h)}\right)}, \tag{24}$$

where $\beta_{2(xh)}$ is the population coefficient of kurtosis of the auxiliary variable for the h^{th} stratum. The Biases of $\bar{y}_{R(S_t MRSSk)3}$, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)3}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\lambda_2{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \lambda_2 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h}\right), \tag{25}$$

$$Bias\left(\bar{y}_{(S_t MRSSE)3}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\lambda_2{}^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right) - \lambda_2 \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right)\right]. \tag{26}$$

The $MSEs$ of $\bar{y}_{R(S_t MRSSk)3}$ for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)3}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \lambda_2{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\lambda_2 \frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right], \tag{27}$$

$$MSE\left(\bar{y}_{R(S_t MRSSE)3}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2} \right) + \lambda_2{}^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) \right]$$
$$-\lambda_2 \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right). \tag{28}$$

iv) If $a_h = \beta_{2(x_h)}$ and $b_h = C_{x_h}$ then from Equation (8), we get

$$\bar{y}_{R(S_t MRSSk)4} = \bar{y}_{[S_t MRSSk]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h \beta_{2(x_h)} + C_{x_h} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(MRSS)} \beta_{2(x_h)} + C_{x_h} \right)}, \tag{29}$$

where $\beta_{2(x_h)}$ and C_{xh} are the population coefficient of kurtosis and coefficient of variation of the auxiliary variable for the h^{th} stratum respectively.

The Biases of $\bar{y}_{R(S_t MRSSk)4}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)4}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\lambda_3{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \lambda_3 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h} \right), \tag{30}$$

$$Bias\left(\bar{y}_{(S_t MRSSE)4}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\lambda_3{}^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) - \lambda_3 \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right) \right]. \tag{31}$$

The *MSEs* of $\bar{y}_{R(S_t MRSSk)4}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)4}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \lambda_3{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\lambda_3 \frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right], \tag{32}$$

$$MSE\left(\bar{y}_{R(S_t MRSSE)4}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right) + \lambda_3{}^2\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right)\right]$$

$$-\lambda_3 \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right). \tag{33}$$

v) If $a_h = C_{x_h}$ and $b_h = \beta_{2(x_h)}$ then from Equation (8), we get

$$\bar{y}_{R(S_t MRSSk)5} = \bar{y}_{[S_t MRSSk]} \frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h C_{x_h} + \beta_{2(x_h)}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(MRSS)} C_{x_h}\right) + \beta_{2(x_h)}}. \tag{34}$$

The Biases of $\bar{y}_{R(S_t MRSSk)5}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)5}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\lambda_4{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \lambda_4 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h}\right), \tag{35}$$

$$Bias\left(\bar{y}_{(S_t MRSSE)5}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\lambda_4{}^2\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right) - \lambda_4\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right)\right]. \tag{36}$$

The $MSEs$ of $\bar{y}_{R(S_t MRSSk)5}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)5}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \lambda_4{}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\lambda_4 \frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right], \tag{37}$$

$$MSE\left(\bar{y}_{R(S_t MRSSE)5}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right) + \lambda_4{}^2\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right)\right]$$
$$-\lambda_4 \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right). \tag{38}$$

6. SECOND PROPOSED CLASS OF ESTIMATORS

Following Al-Omari [1], we proposed an other class of ratio-type estimators in $S_t MRSS$, given by

$$\bar{y}_{(S_t MRSSk)G} = \bar{y}_{[S_t MRSSk]}\left[\omega \frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + q_{1h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(MRSS)} + q_{1h}\right)} + (1-\omega)\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + q_{3h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(MRSS)} + q_{3h}\right)}\right], \tag{39}$$

where ω is scalar quantity and q_{1h} and q_{3h} are the first and third quartiles of auxiliary variable in the h^{th} stratum respectively.

In terms of $\xi's$, we have

$$\bar{y}_{(S_t MRSSk)G} = \bar{Y}\left(1+\xi_0\right)\left(1+\eta_1\xi_1\right)^{-1}\left(1+\eta_2\xi_1\right)^{-1},$$

or

$$\bar{y}_{(S_t MRSSk)G} - \bar{Y} = \bar{Y}\left[\begin{array}{l}\xi_0 - \left\{\eta_2 + \omega(\eta_1 - \eta_2)\right\}\xi_1 \\ \quad + \left\{\eta_2^2 + \omega(\eta_1^2 - \eta_2^2)\right\}\xi_1^2 - \left\{\eta_2 + \omega(\eta_1 - \eta_2)\right\}\xi_0\xi_1\end{array}\right],$$

where $\eta_1 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + q_{1h}\right)}$, $\eta_2 = \dfrac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h\left(\bar{X}_h + q_{3h}\right)}$.

The Biases of $\bar{y}_{(S_t MRSSk)G}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)G}\right) \cong \bar{Y}\left\{\eta_2^2 + \omega\left(\eta_1^2 - \eta_2^2\right)\right\} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} \right)$$

$$-\bar{Y}\left\{\eta_2 + \omega\left(\eta_1 - \eta_2\right)\right\} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h} \right) \quad (40)$$

and

$$Bias\left(\bar{y}_{(S_t MRSSE)G}\right) \cong \bar{Y}\left\{\eta_2^2 + \omega\left(\eta_1^2 - \eta_2^2\right)\right\} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right)$$

$$-\bar{Y}\left\{\eta_2 + \omega\left(\eta_1 - \eta_2\right)\right\} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right). \quad (41)$$

The *MSEs* of $\bar{y}_{(S_t MRSSk)G}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)G}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left[\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \left\{\eta_2 + k(\eta_1 - \eta_2)\right\}^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} \right]$$

$$-2\bar{Y}^2 \left\{\eta_2 + \omega(\eta_1 - \eta_2)\right\} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma_{yx_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right), \quad (42)$$

and

$$MSE\left(\bar{y}_{(S_t MRSSE)G}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2} \right) + \left\{ \eta_2 + \omega(\eta_1 - \eta_2) \right\}^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) \right]$$

$$-\bar{Y}^2 \left\{ \eta_2 + \omega(\eta_1 - \eta_2) \right\} \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right). \quad (43)$$

The optimum value of ω is given by

$$\omega_{opt} = \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \frac{\sigma_{y_h}}{\sigma_{x_h}} \rho_{y_h x_h}$$

i) For $\omega = 1$ in Equation (39), we get

$$\bar{y}_{(S_t MRSSk)6} = \bar{y}_{[S_t MRSSk]} \frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h + q_{1h} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_{h(MRSS)} + q_{1h} \right)}. \quad (44)$$

The Biases of $\bar{y}_{(S_t MRSSk)6}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)6}\right) \cong \sum_{h=1}^{L} \frac{W_h^2}{m_h r} \left(\eta_1^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \eta_1 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h} \right) \quad (45)$$

and

$$Bias\left(\bar{y}_{(S_t MRSSE)6}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r} \left[\eta_1^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2} \right) - \eta_1 \left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h} \right) \right]. \quad (46)$$

The *MSEs* of $\bar{y}_{(S_t MRSSk)6}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)6}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \eta_1^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\eta_1 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h}\right) \tag{47}$$

$$MSE\left(\bar{y}_{(S_t MRSSE)6}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right) + \eta_1^2\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right)\right]$$
$$-\eta_1 \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right). \tag{48}$$

ii) For $\omega = 0$ in Equation (39), we get

$$\bar{y}_{(S_t MRSSk)7} = \bar{y}_{[S_t MRSSk]} \frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + q_{3h}\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_{h(MRSS)} + q_{3h}\right)}. \tag{49}$$

The Biases of $\bar{y}_{(S_t MRSSk)7}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$Bias\left(\bar{y}_{(S_t MRSSO)7}\right) \cong \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\eta_2^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - \eta_2 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h}\right), \tag{50}$$

and

$$Bias\left(\bar{y}_{(S_t MRSSE)7}\right) \cong \bar{Y} \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\eta_2^2\left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right) - \eta_2\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right)\right]. \tag{51}$$

The $MSEs$ of $\bar{y}_{(S_t MRSSk)7}$, upto first order of approximation, for odd and even sample sizes are respectively, given by

$$MSE\left(\bar{y}_{(S_t MRSSO)7}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma^2_{y_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h^2} + \eta_2^2 \frac{\sigma^2_{x_h\left(\frac{m_h+1}{2}\right)}}{\bar{X}_h^2} - 2\eta_2 \frac{\sigma_{y_h x_h\left(\frac{m_h+1}{2}\right)}}{\bar{Y}_h \bar{X}_h}\right), \tag{52}$$

$$MSE\left(\bar{y}_{(S_t MRSSE)7}\right) \cong \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{2m_h r}\left[\left(\frac{\sigma^2_{y_h\left(\frac{m_h}{2}\right)} + \sigma^2_{y_h\left(\frac{m_h+2}{2}\right)}}{\bar{Y}_h^2}\right) + \eta_1^2 \left(\frac{\sigma^2_{x_h\left(\frac{m_h}{2}\right)} + \sigma^2_{x_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h^2}\right)\right]$$

$$-\eta_1 \bar{Y}^2 \sum_{h=1}^{L} \frac{W_h^2}{m_h r}\left(\frac{\sigma_{yx_h\left(\frac{m_h}{2}\right)} + \sigma_{yx_h\left(\frac{m_h+2}{2}\right)}}{\bar{X}_h \bar{Y}_h}\right). \tag{53}$$

Table 1
***PREs* of First Class of Estimators When Sample Size is Odd**

$L=3$, $\mu_{xh}=(2,3,4)$, $\mu_{yh}=(3,4,6)$, $\sigma_{yh}=(1,1,1)$, $\sigma_{xh}=(1,1,1)$, $W_h=(.30,.30,.40)$, $m_h=(5,5,5)$, and $r=3$.					
ρ_{yxh}	$PRE(1)$	$PRE(2)$	$PRE(3)$	$PRE(4)$	$PRE(5)$
0.95, 0.95, 0.95	178.90	178.98	179.32	178.37	179.29
0.90, 0.90, 0.90	149.87	150.06	150.78	149.70	151.40
0.70, 0.70, 0.70	116.15	116.24	116.75	116.72	116.91
0.50, 0.50,0.50	110.28	110.34	110.53	110.07	111.06
-0.95,-0.95, -0.95	135.29	135.17	134.66	135.61	134.24
-0.90,-0.90, -0.90	115.90	115.95	116.15	116.08	116.42
-0.70,-0.70, -0.70	110.55	110.53	110.42	110.41	110.83
-0.50,-0.50, -0.50	105.98	105.85	105.37	105.90	105.02
0.95, 0.90, 0.70	157.16	157.38	158.41	156.52	158.44
0.90, 0.70, 0.50	115.45	115.78	117.29	116.38	118.09
0.70, 0.50, 0.30	107.39	107.33	107.13	107.42	107.48
-0.99,-0.90, -0.70	126.53	126.42	125.95	126.60	125.65
-0.90, -0.70, -0.50	111.46	111.2	110.55	111.44	110.20
-0.70,-0.50, -0.30	103.02	102.88	102.87	102.77	102.30

Table 2
PREs of First Class of Estimators When Sample Size is Even

$L=3$, $\mu_{xh}=(2,3,4)$, $\mu_{yh}=(3,4,6)$, $\sigma_{yh}=(1,1,1)$, $\sigma_{xh}=(1,1,1)$, $W_h=(.30,.30,.40)$, $m_h=(4,4,4)$, and $r=3$.					
ρ_{yxh}	$PRE(1)$	$PRE(2)$	$PRE(3)$	$PRE(4)$	$PRE(5)$
0.95, 0.95, 0.95	179.06	179.37	180.83	177.40	181.48
0.90, 0.90, 0.90	139.92	140.12	141.07	140.09	141.62
0.70, 0.70, 0.70	108.37	108.56	109.57	108.75	110.84
0.50, 0.50,0.50	102.72	102.88	103.67	102.41	104.16
-0.95,-0.95, -0.95	145.13	143.32	144.93	142.91	145.58
-0.90,-0.90, -0.90	120.09	120.11	120.17	120.36	120.56
-0.70,-0.70, -0.70	115.35	115.43	115.76	115.33	115.96
-0.50,-0.50, -0.50	105.64	105.71	106.02	105.81	106.35
0.95, 0.90, 0.70	158.86	158.98	159.56	158.80	160.22
0.90, 0.70, 0.50	123.68	123.72	123.93	123.53	124.05
0.70, 0.50, 0.30	102.62	102.69	103.56	103.01	103.65
-0.99,-0.90, -0.70	118.40	118.75	120.16	118.82	121.75
-0.90, -0.70, -0.50	110.06	110.17	110.60	110.19	110.90
-0.70,-0.50, -0.30	101.10	101.07	101.02	101.17	101.18

Table 3
PREs of Second Class of Estimators When Sample Size is Odd

$L=3$, $\mu_{xh}=(2,3,4)$, $\mu_{yh}=(3,4,6)$, $\sigma_{yh}=(1,1,1)$, $\sigma_{xh}=(1,1,1)$, $W_h=(.30,.30,.40)$, $m_h=(5,5,5)$, and $r=5$.			
ρ_{yxh}	$PRE(6)$	$PRE(7)$	$PRE(G)$
0.95, 0.95, 0.95	150.69	150.88	171.24
0.90, 0.90, 0.90	116.67	116.83	134.11
0.70, 0.70, 0.70	110.54	110.62	124.56
0.50, 0.50,0.50	104.19	104.26	121.20
-0.95,-0.95, -0.95	134.62	134.73	151.01
-0.90,-0.90, -0.90	116.13	116.19	133.77
-0.70,-0.70, -0.70	110.38	110.41	123.31
-0.50,-0.50, -0.50	105.30	105.44	117.66
0.95, 0.90, 0.70	158.30	158.60	176.08
0.90, 0.70, 0.50	117.09	117.66	134.42
0.70, 0.50, 0.30	107.09	107.14	122.91
-0.99,-0.90, -0.70	125.83	125.98	143.07
-0.90,-0.70, -0.50	110.37	110.66	129.33
-0.70,-0.50, -0.30	102.26	102.48	114.26

Table 4
PREs of Second Class of Estimators When Sample Size is Even

$L = 3$, $\mu_{xh} = (2,3,4)$, $\mu_{yh} = (3,4,6)$, $\sigma_{yh} = (1,1,1)$, $\sigma_{xh} = (1,1,1)$, $W_h = (.30,.30,.40)$, $m_h = (4,4,4)$, and $r = 5$.			
ρ_{yxh}	$PRE(6)$	$PRE(7)$	$PRE(G)$
0.95, 0.95, 0.95	181.51	181.63	201.27
0.90, 0.90, 0.90	141.69	141.74	166.30
0.70, 0.70, 0.70	110.88	110.92	134.01
0.50, 0.50,0.50	104.19	104.26	121.20
-0.95,-0.95, -0.95	145.61	145.69	169.87
-0.90,-0.90, -0.90	120.64	120.77	141.00
-0.70,-0.70, -0.70	115.98	116.04	137.83
-0.50,-0.50, -0.50	106.41	106.52	119.52
0.95, 0.90, 0.70	160.27	160.33	183.09
0.90, 0.70, 0.50	124.11	124.18	143.11
0.70, 0.50, 0.30	103.69	103.76	118.70
-0.99,-0.90, -0.70	121.78	121.83	139.61
-0.90,-0.70, -0.50	110.94	110.97	132.38
-0.70,-0.50, -0.30	101.18	101.23	116.26

7. SIMULATION STUDY

To compare the performances of the proposed classes of estimators, a simulation study is conducted where ranking is performed on the auxiliary variable X. Bivariate random observations $(X_{(i)h}, Y_{[i]h})$, $i = 1,2,...,m_h$; and $h = 1,2,...,L$ are generated from a bivariate normal population having parameters $\left(\mu_{xh}, \mu_{y_h}, \sigma_{x_h}, \sigma_{y_h}, \rho_{yx_h}\right)$. Using $20,000$ simulations, estimates of $MSEs$ for ratio-type estimators are computed under $S_t RSS$ and $S_t MRSS$. Estimators are compared in terms of Percent Relative Efficiencies $(PREs)$. We used the following expressions to obtain the $PREs$:

$$PRE(p) = \frac{MSE\left((\bar{y}_{R(S_t RSS)p}\right)}{MSE\left(\bar{y}_{R(S_t MRSSk)p}\right)} \times 100,\ k = (O, E),\ p = (1,2,...5)$$

$$PRE(s) = \frac{MSE\left(\bar{y}_{R(S_t RSS)1}\right)}{MSE\left(\bar{y}_{(S_t MRSSk)s}\right)} \times 100,\ k = (O, E),\ s = (6,7,G)$$

The $PREs$ of proposed classes of estimators using $S_t MRSS$ in comparison with different stratified ranked set estimators for odd and even sample sizes are shown in Tables 1, 2, 3 and 4 respectively.

The simulation results showed that with decrease of the correlation coefficients ρ_{yx_h}, $PREs$ decreases which are expected results. The numerical values given in the first eight rows are obtained by assuming equal correlations across the strata whereas the last six rows assume unequal correlations across the strata. It is much easy to conclude from the results given in Tables 1 to 4 that our proposed estimators perform much better than their competitors.

8. NUMERICAL ILLUSTRATION

To observe performances of the estimators, we used the following real data set.

Population [Source: Singh [18]]

The study variable y and the auxiliary variable x are defined below.

y: The Tobacco production in metric tons,

x: The area for Tobacco in specified countries during 1998.

Table 5
Summary Statistics

Stratum 1	**Stratum 2**	**Stratum 3**
$N_1 = 12$	$N_2 = 30$	$N_3 = 17$
$m_1 = 3$	$m_2 = 5$	$m_3 = 3$
$n_1 = 9$	$n_2 = 15$	$n_3 = 9$
$W_1 = 0.0234$	$W_2 = 0.5085$	$W_3 = 0.02881$
$\bar{X}_1 = 5987.83$	$\bar{X}_2 = 11682.73$	$\bar{X}_3 = 68662.29$
$\bar{Y}_1 = 11788$	$\bar{Y}_2 = 16862.27$	$\bar{Y}_3 = 227371.53$
$\bar{R}_1 = 1.97$	$\bar{R}_2 = 1.44$	$\bar{R}_3 = 3.31$
$S_{x_1}^2 = 27842810.5$	$S_{x_2}^2 = 760238523$	$S_{x_3}^2 = 12187889050$
$S_{y_1}^2 = 1538545883$	$S_{y_2}^2 = 2049296094$	$S_{y_3}^2 = 372428238550$
$S_{y_1x_1} = 62846173.1$	$S_{y_2x_2} = 1190767859$	$S_{y_3x_3} = 27342963562$
$C_{x_1} = 0.8812$	$C_{x_2} = 2.3601$	$C_{x_3} = 1.6079$
$\beta_{2(x1)} = 14.6079$	$\beta_{2(x2)} = 10.7527$	$\beta_{2(x3)} = 8.935$
$\rho_{yx1} = 0.9602$	$\rho_{yx2} = 0.9540$	$\rho_{yx3} = 0.4058$

From above population, we draw median ranked set samples of odd and even sample sizes from stratum 1^{st}, 2^{nd} and 3^{rd} respectively. Further each median ranked set sample from each stratum is repeated with number of cycles r. Hence sample sizes of stratified median ranked set samples equivalent to stratified ranked set sample of sizes $n_h = m_h r$. The estimated $PREs$ based upon $MSEs$ values of various stratified median ranked set estimators in comparison with different stratified ranked set estimators are shown in Table 6. It showed that our proposed ratio-type estimators under S_tMRSS are more efficient than their competitors in S_tRSS.

Table 6
PREs of Different Estimators using Real Data Set

Sample size	$PRE(1)$	$PRE(2)$	$PRE(3)$	$PRE(4)$	$PRE(5)$	$PRE(6)$	$PRE(7)$	$PRE(G)$
Odd	168.09	168.19	169.43	168.67	169.19	169.38	169.43	186.30
Even	169.10	169.57	170.73	167.14	171.66	171.53	171.07	191.12

9. CONCLUSION

In this study, we proposed two different classes of ratio-type estimators in S_tMRSS to estimate the finite population mean by adopting the Mandowara and Mehta [13] and Al-Omari [1] estimators. The Biases and $MSEs$ of these proposed estimators are derived up to first order of approximation. Both simulation and empirical studies are conducted to observe the performances of estimators. On the basis of simulation study and numerical illustration, our proposed ratio-type estimators under S_tMRSS performed better as compared to respective competitive estimators in S_tRSS. Also, among all estimators $\bar{y}_{(S_tMRSSk)G}$ is more efficient.

ACKNOWLEDGMENT

The authors are thankful to the chief editor and the anonymous referees for their valuable comments which helped to improve the research paper.

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