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A NEW APPROACH FOR EXTREME VALUES IN DATA ENVELOPMENT ANALYSIS

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ABSTRACT

Data Envelopment Analysis (DEA) is methodology for relative performance measurement and has been utilised extensively over the past few decades. DEA scores are however sensitive when outliers are present in the data and can cause inaccurate reflections of the relative efficiency score and the projections of inefficient Decision Making Units (DMU) onto the efficient frontier. Stochastic frontier analysis can accommodate for the statistical noise but makes certain assumptions on the data. This study introduces an approach to accommodate for outliers in a DEA model without removing observation that would otherwise affect the results. The results on the proposed model are compared to two deterministic and three stochastic models, and have shown an increase in the efficiency and the number of efficient DMUs and an increase in the overall efficiency scores.

KEYWORDS

Data Envelopment Analysis, Stochastic Frontier Analysis, Outliers

1. INTRODUCTION

DEA is methodology of measuring the relative efficiency of Decision Making Units (DMU) using several variables in form of inputs and outputs. This method was popularised in 1978 and has been widely applied to various industries from banking, hospital, libraries to municipalities. The core concept of DEA is that provides an efficiency score of a DMU relative to the remaining DMUs in the reference set. This method works well when the input and output variables are homogenous [(Charnes et al. (1978)].

In DEA outliers in the inputs and outputs can result in inaccurate measures of the relative efficiency scores. Since DEA is a methodology of relative efficiency, it is very sensitive to outliers. Outliers are atypical observations resulting from recording errors and should be replaced with the correct value or alternatively deleted from the data [Wilson (1993)]. It is possible for a sample to have data points which arise from a data-generating process (DGP) whereby the value has a very low probability of occurring. These data points would appear as outliers but one would not expect a high frequency of occurrence [Wilson (1993)].

Given the nature of non-parametric frontier models, such as DEA, requiring few assumptions they become very appealing yet are very sensitive to extreme values. An outlier in a DMU can cause other DMU efficiency scores to be over or under estimated. The result of the contaminated values leads to an efficient frontier that is not a true reflection of the efficiency scores. Projections of the inefficient DMU onto the efficient frontier will also be inaccurate. Therefore detecting outliers is an important step but is not easily done in a multivariate setup [Simar (2003)].

The need for identifying and eliminating outliers in DEA was pointed out by [Simar (1996)]. If an outlier or extreme value exists in the data, it can influence the efficiency score of the other DMUs. Simar (1996) stated that one approach to dealing with outlier is to make use of a stochastic frontier production model which was introduced independently by Aigner and Chu (1968). Aigner and Chu (1968) and Meeusen and Broeck (1977) which accommodated for the noise in the data. It calculated an efficiency score and an error term to reflect random noise. Various authors have developed models which can in the process of calculating the efficiency score identify outliers and influential observations [Al-Refaie, et al. (2014), Andersen and Petersen (1993), Diabat et al. (2015), Wilson (1995), Simar (2003), Johnson and McGinnis (2008) and Banker and Chang (2006)], to name a few.

Wilson (1993) argued that due to the lack of a stochastic structure, DEA models are not susceptible to standard outlier detection methods in linear regression models by using the Ordinary Least Squares (OLS) residuals. There have been numerous studies which have emphasized the need for outlier detection in DEA [Banker and Chang (2006), Banker and Gifford (1988), Datar et al. (1990), Johnson and McGinnis (2008), Tan et al. (2015), Timmer (1971) and Wilson (1993)] to name a few.

This study proposes an approach to accommodate for extreme values in the input variables and output variables of a DEA model. This study applies the approach to three populations of municipalities in South Africa and compares the results to two deterministic DEA models and three Stochastic Frontier Analysis models. This rest of the paper is structured as follows: Section 2 provides an overview of the deterministic DEA models, the Stochastic Frontier Analysis approach and the outlier detection models developed over the years with a review of the literature. Section 3 provides details on the proposed approach to accommodate for extreme values in a DEA analysis. Section 4 discusses the data used for the model and a literature review of past studies into municipalities using both DEA and SFA. Section 5 contains the results of the comparison and an interpretation of the findings. Section 6 concludes the paper and discusses future research.

2. METHODOLOGIES

2.1 Overview of DEA

DEA is a mathematical programming procedure, developed by Charnes et al. (1978) that can be used to measure the relative efficiency of DMU. Each production unit of a set of comparable producing unit can be regarded as a DMU. DEA aims at evaluating the relative efficiencies of entities in the sample and provides a measure of relative performance rather than comparing it with an idealized benchmark or level of performance.

The basic DEA model is a Charnes et al. (1978) model or CCR model which assumes constant returns to scale (CRS). Banker et al. (1984) proposed a model (BCC) which assumes variables return to scale (VRS). This was accomplished by adding a constraint where all the sum of the weights equals one.

The core concept of measuring efficiency can be dated back to Farrell (1957). DEA evaluates the relative efficiency of homogenous DMU's which have no known relationship between the incorporation of inputs used and the outputs generated from the unit. The vital characteristic of DEA is the ability to transform a multi-input and multi-output unit into a single unit of measurement. However DEA can formulate the relative efficiency of any number of DMU's. The relative efficiency of DMUs can be calculated in relation to all other DMUs.

The objective function in the conventional DEA model is to maximize the relative efficiency scores, subject to the set of weights required for each DMU, which must be feasible. The variables for any DEA model are as follows. Assume that there are *n* DMUs to be analyzed, each DMU has *m* inputs which contribute to the *s* outputs produced. DEA assigns a set of weights to the inputs and outputs of a DMU with the aim of yielding the best possible efficiency.

The decision variables for the model are the set of weights assigned to each input and output. Let:

x_{ij} : amount of input *i* used for DMU *j*.
y_{rj} : amount of output *r* which is produced by DMU *j*.
v_i : calculated weight assigned to input *i*
u_r : calculated weight assigned to output *r*.

Table 1 below gives an overview of the DEA models, with the objective function and constraints.

Table 1
Deterministic DEA models

	CCR Model	**BCC Model**
Objective Function	$\max\ h_o(u,v) = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$	$\max\ h_o(u,v) = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$
Subject To	$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1\ \forall\, j \in \{1,..,n\}$ $u_r \geq 0\ \forall\, r \in \{1,..,s\}$ $v_i \geq 0\ \forall\, i \in \{1,..,m\}$	$\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1\ \forall\, j \in \{1,..,n\}$ $u_r \geq 0\ \forall\, r \in \{1,..,s\}$ $v_i \geq 0\ \forall\, i \in \{1,..,m\}$ $\sum_{j=1}^{n} u_{\mathrm{j}} = 1$

The h_o value is a ratio calculated of the weighted sum of outputs divided by the weighted sum of inputs, such that it is maximized for DMU_o the DMU to be evaluated.

DEA can be seen as a non-statistical and non-parametric approach to production frontier and efficiency analysis. The approach is non-statistical because the estimates not are based on any statistical distribution, furthermore the ability to detect noise in not explicitly considered Fare et al. (2001). The non-parametric characteristics of DEA imply that there is no assumption needed to define a functional relationship between the inputs and the outputs i.e. no statistical distributions assumptions are made and estimates are based on assumed statistical distributions [Fare et al. (2001)]. Another characteristic of DEA is that the efficiency score for the DMUs are calculated using different weights. This supports the criticism of DEA being non-statistical as the resulting efficiency scores cannot be easily validated using traditional statistical methods [Diabat et al. (2015) and Fare et al. (2001)].

2.2 Overview of Stochastic Frontier Analysis

One of the approaches to overcome the effect of outliers in an efficiency analysis is to make use of a Stochastic Frontier Analysis. An advantage of the Stochastic Frontier Analysis (SFA) is in its ability to differentiate between the random noise in the calculations and the estimated production frontier.

The econometric approach involved developing a SFA model is founded on the deterministic parameter frontier which was proposed by Aigner and Chu (1968). These frontier production models were introduced independently by Aigner et al. (1977) and Leys et al. (2013). Once the functional form has been chosen for the development of the production frontier, the authors proposed the model.

$$y_i = f(x_i, \beta) + \varepsilon_i$$

In the equation above y_i is denoted as the output obtained by DMUi, x_i is the vector of selected inputs. Beta is the vector of parameters to be estimated and ε_i is the composed error term. The composite error term comprises of the two elements: $\varepsilon_i = v_i + u_i$, where v_i represents the symmetric disturbance that encapsulate the random variation in the production frontier. Random error is one factor that is attributed to the disturbance. Other factors which contribute to the disturbance are errors in observations and measuring of data and chance. All of these disturbances are assumed to be identically and independently distributed following a $N\sim(0, \sigma^2)$ distribution. The error component u_i is an asymmetric and encapsulate the technical inefficiency. This component is assumed to be distributed independently of v_i and satisfies the condition $u_i \leq 0$. The statistical distribution of error component u_i is assumed and the cases for a half-normal and exponential distribution was analyzed [Leys et al. (2013)], and only considered the exponential distribution. This model assumes that the production function takes a log-linear Cobb-Douglas form hence the stochastic frontier production model can be written as:

$$\ln(y_i) = \beta_0 + \sum_n \beta_n \ln(x_{ni}) + \varepsilon_i$$

Table 2 below provides the distribution, assumptions, joint density function, marginal density function and Log-likelihood function for the production models under the three error terms used in this study.

Table 2
Stochastic Frontier Model Equations

Error Term Distribution	Assumptions	Joint Density Function	Marginal Density Function	Log-likelihood function for the production model
Normal-Half Normal Model	v_i *is iid with* $N\ (0,\sigma_v^2)$ u_i *is iid with* $N^+(0,\sigma_u^2)$ v_i*and* u_i *independent of each other, and of the regresso*	$f(u,v) = \frac{2}{2\pi\sigma_u\sigma_v} exp\left\{-\frac{u^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right\}$	$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)du$ $= \frac{2}{\sqrt{2\pi\sigma}}\left[\mathrm{I} - \phi\left(\frac{\varepsilon\lambda}{\sigma}\right)\right] exp\left\{-\frac{\varepsilon^2}{2\sigma^2}\right\}$ $= \frac{2}{\sigma}\phi\left(\frac{\varepsilon}{\sigma}\right)\Phi\left(-\frac{\varepsilon\lambda}{\sigma}\right)$ where $\lambda = \frac{\sigma_u}{\sigma_v}$ and $\sigma = \sqrt{\sigma_u^2+\sigma_v^2}$	$lnL = constant - Nln\sigma + \sum_i ln\Phi\left(-\frac{\varepsilon_i\lambda}{\sigma}\right) - \frac{1}{2\sigma^2}\sum_i \varepsilon_i^2$
Normal-Exponential Model	v_i*is iid* $N(0,\sigma_v^2)$ u_i*is iid exponential* v_i*and* u_i *independent of each other, and of the regresso*	$f(u,v) = \frac{1}{\sqrt{2\pi}\,\sigma_u\sigma_v} exp\left\{-\frac{u}{\sigma_u} - \frac{v^2}{2\sigma_v^2}\right\}$	$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)du$ $= \left(\frac{1}{\sigma_u}\right)\Phi\left(-\frac{\varepsilon}{\sigma_v} - \frac{\sigma_v}{\sigma_u}\right) exp\left\{\frac{\varepsilon}{\sigma_u} + \frac{\sigma_v^2}{2\sigma_u^2}\right\}$	$lnL = constant - Nln\sigma_u + N\left(\frac{\sigma_v^2}{2\sigma_u^2}\right) + \sum_i \frac{\varepsilon_i}{\sigma_u} + \sum_i ln\Phi\left(\frac{\varepsilon_i}{\sigma_v} - \frac{\sigma_v}{\sigma_u}\right)$
Normal-Truncated Normal Model	v_i *is iid with* $N\ (0,\sigma_v^2)$ u_i *is iid with* $N^+(u,\sigma_u^2)$ v_i*and* u_i *independent of each other, and of the regressor*	$f(u,v) = \frac{1}{\sqrt{2\pi}\,\sigma_u\sigma_v\,\Phi\left(\frac{\mu}{\sigma_u}\right)} exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right\}$	$f(\varepsilon) = \int_0^\infty f(u,\varepsilon)du$ $= \frac{1}{\sqrt{2\pi}\sigma\Phi(\frac{\mu}{\sigma_u})}\Phi\left(\frac{\mu}{\sigma\lambda} - \frac{\varepsilon\lambda}{\sigma}\right) exp\left\{-\frac{(\varepsilon+\mu)^2}{2\sigma^2}\right\}$ $= \frac{1}{\sigma}\phi\left(\frac{\varepsilon+\mu}{\sigma}\right)\Phi\left(\frac{\mu}{\sigma\lambda} - \frac{\varepsilon\lambda}{\sigma}\right)\left[\Phi\left(\frac{\mu}{\sigma_u}\right)\right]^{-1}$	$lnL = constant - Nln\sigma - Nln\Phi\left(\frac{\mu}{\sigma_u}\right) + \sum_i ln\Phi\left(\frac{\mu}{\sigma\lambda} + \frac{\varepsilon_i\lambda}{\sigma}\right) - \frac{1}{2}\sum_i\left(\frac{\varepsilon_i+\mu}{\sigma}\right)^2$

Since the random error u_i cannot be directly observed, it poses a problem with the above mentioned models. The suggestion that overcame this problem was given by Huber (1981) who estimated the inefficiency using a conditioned distribution of u_i given ε_i . The specification of the distribution of u_i given it is exponetial is as follows:

$$E(u_i|\varepsilon_i) = \sigma_v \left\{ \frac{f^* \left[\frac{\left(\frac{\varepsilon_i}{\sigma_v}\right)}{\left(\frac{\sigma_v}{\phi}\right)} \right]}{1 - F^* \left[\frac{\left(\frac{\varepsilon_i}{\sigma_v}\right)}{\left(\frac{\sigma_v}{\phi}\right)} \right]} - \left[\frac{\left(\frac{\varepsilon_i}{\sigma_v}\right)}{\left(\frac{\sigma_v}{\phi}\right)} \right] \right\}$$

where f^* is the standard normal density. The equation denotes F^* is the cumulative distribution functions. The efficiency of each observation can be estimated as $\exp[-E(u_i|\varepsilon_i)]$ only when $E(u_i|\varepsilon_i)$ is known [Jondrow et al. (1982)].

The main difference between SFA and DEA is that the SFA creates a stochastic frontier together with a probability distribution whereas DEA is a completely non-stochastic frontier. DEA analysis uses a non-parametric approach to produce an efficiency frontier. It does not involve imposing any assumptions on the functional form i.e. it does not take into consideration the influence of external factors. It is a non-statistical approach which disregards statistical noise but because it is non-parametric, there are few underlying assumptions that need to be taken. On the other hand SFA takes into account the statistical noise but then it then requires certain assumptions to be made.

2.3 Outliers Detection Models

Outliers in DEA models can provide unwanted noise and inaccurate efficiency score. Some researchers have developed methods to remove the k most influential observations in from the model to eliminate the outliers. However DEA is a methodology to provide the relative efficiency scores if one removes the observations with the outlier then it will change the outcome of the model thus providing an inaccurate reflection of the scores. Other researchers have developed new methods to detect the outliers as part of the DEA model.

If an outlier was present in an observation, Timmer (1971) suggested removing the observation from the reference set and calculating the frontier on the remaining DMUs. This approach will provide an accurate representation of the efficiency score but will not be reflective of the entire sample of DMUs (outlier DMU included). By removing the outlier DMU the remaining DMUs score will inherently change as they will not be contaminated or influenced by the outlier observation.

Wilson (1993) developed a different outlier detection method in non-parametric models and was further improved upon by Wilson (1995) by using the super-efficiency model of [6]. A new method to detect influential observation was introduced by Wilson (1995).

Andersen and Petersen (1993) used a super-efficiency model to which involved using the conventional DEA model. This was accomplished by removing the DMU which was being evaluated from the reference set. The conventional DEA model calculates the efficiency of a DMU relative to a reference set of all the observations including its own observation. A super-efficiency model removes itself from the reference set allowing for an efficiency score that exceeds one [Banker et al. (1984)]. The model used by Andersen and Petersen (1993) was based on the super-efficiency model proposed by Banker and Chang (2006).

Simar (2003) introduced another method for identifying influential observations and extended this to both input and output orientations. Banker and Chang (2006) performed simulated experiments with the aim of evaluating the performance of super efficiency models of Banker and Chang (2006) when it is used to rank the efficient unit and outlier detection. Banker and Chang (2006) provided an explanation on how the model by Andersen and Petersen (1993) can be used for outlier detection and it not suitable for the ranking of these units.

Johnson and McGinnis (2008) introduced semi-parametric DEA model which utilizes two-stages for the detection of outliers which involved constructing an inefficient frontier to detect outliers. According to Yang et al. (2010), Banker and Chang (2006) and Johnson and McGinnis (2008) showed evidence that the use of super efficiency DEA approaches can lead to detecting outliers. Yang et al. (2010) stated that both these studies lacked the comparison of approaches to popular statistical and data mining techniques for outlier detection. Furthermore, these studies did not state the required conditions for successful detection of outliers and they did not explore the potential predictive capabilities and performance of the outlined approaches [Yang et al. (2010)].

The need for identifying and eliminating outliers in DEA was pointed out by Simar (1996). Simar (1996) further pointed out that if outliers cannot be identified a stochastic frontier is recommended. Leys et al. (2013) proposed a method of removing the k most influential DMUs using an empirical specification tests. Tran et al. (2010) proposed a new method of dealing with outliers in DEA based on two scalar measures. These measures are the relative frequency where an observation occurs in the frontier and the second measure is the cumulative weight of the observation in the frontier. Cazals et al. (2002) introduced robust nonparametric efficiency score which had the ability to cater for outliers. It is based on a concept of expected minimum /maximum function. For this study proposes a new model to cater for outliers in a DEA analysis by substituting specific input and outputs which are considered outliers. Table 3 below, shows an overview of the literature around outlier detection model and approaches over the years.

Table 3
Literature Review of Outlier Studies, Treatment of Outliers in DEA

Authors	Methodology	Approach and Findings
Timmer 1971	Probabilistic frontier production frontier using a Cobb-Douglas production frontier	Procedure of removing fixed percentage of the observations until the estimate the production frontier stabilized
Andersen and Petersen 1993	DEA super-efficiency	Proposed modified approach to allow for ranking of efficient units and detect influential observations in the sample. Based on the super-efficiency model.

Authors	Methodology	Approach and Findings
Wilson 1993	DEA	Extended the statistic of Andersen and Petersen (1993) to cater for multiple outputs. Found that although an observation had a low probability of occurrence, it is not conclusive that it may be an outlier.
Wilson 1995	DEA	Improvements were made to the initial method developed in Wilson (1993). The super efficiency models used in Andersen and Petersen (1993) was adopted. The improvement of the model allows for a less computationally expensive approach
Simar and Wilson 1998	DEA with Bootstrapping	Proposed a bootstrap strategy focused around a reasonable Data-Generating Process and approximate the sampling variation of the estimated frontier.
Simar and Wilson 2000	DEA with Bootstrapping	Extended the work of Simar and Wilson 1998. The proposed method alleviated the restrictive method in Simar and Wilson 1998 by allowing for heterogeneity in the structure of efficiency
Cazals Florens and Simar 2002	Non-Parametric estimator based on the expected minimum function or maximum output function	The approach proposed in the paper is related to DEA/FDH estimators of efficiency but is more robust to outliers, noise and extreme values. The proposed idea is based on the concept of ‘‘expected frontier of order-m’’
Simar 2003	Non-parametric frontier model	Based on the work of Cazals, Florens and Simar (2002), Simar (2003) demonstrates how the procedure can be used to detect outliers. This approach is multivariate and can be applied to a DEA/FDH approach
Banker and Chang 2006	DEA, Super efficiency	Researched two alternative uses of the super efficiency procedure. The first was in detecting outliers and the second was for ranking efficient DMUs.
Johnson and McGinnis 2008	Two-stage semi parametric DEA. Super efficiency and bootstrapping	Introduced the method of using the efficient frontier as well as the inefficient frontier in an attempt to identify outliers. This super-efficiency method implementation used an iterative outlier detection approach and incorporated a semi-parametric bootstrapping method.

Authors	Methodology	Approach and Findings
Tran, Shively and Preckel 2010	DEA	Proposed a new method of detecting outliers based on two scalar measures. The first of these measures, is calculated when testing the efficiency of other observations. This is the relative frequency with which an observation occurs in the efficient frontier. The cumulative weight of an observation in the frontier is used as the second measure.
Chen and Johnson 2010	DEA	Identified a set of axioms and developing an approach consistent with the axioms. This approached allowed for detection of both efficient and inefficient outliers that would have otherwise influences post analysis procedures.
Yang, Wang and Sun 2010	DEA and bootstrapping	The proposed approach introduced two parameters, probability level and tolerance. Both parameters must be specified externally. A bootstrap was also proposed to approximate the true distribution.
Bellini 2012	DEA super-efficiency and forward search	The approach proposed in this papers merged a super efficiency model with a forward search and introduced a distance to be monitored along the search. The distance was obtained by integrating a regression model with the super efficiency DEA model
Bahari and Emrouznejad 2014	DEA , bootstrapping	An alternative approach to Yang et al (2010) is proposed and is applied to a sample of hospitals.
Yang, Wang Zheng 2014	Bi-super DEA, Predictive DEA	To detect outliers, a super DEA based method was adopted. This method constructed both frontiers. A predictive DEA method is also proposed to address the performance of the method. This study used simulated experiments with the aim of examining the performance with regards to the outlier detection methods.

3. PROPOSED MODEL

The proposed model in this paper harnesses standard techniques and identifies outliers in the data and then is accommodated for in the model. The model uses the predetermined identified outliers and substitutes the outliers when calculating the score for a particular DMU. In doing this the score using this approach of the DMU being calculated is not influenced by the outlier. When the score for the DMU with the outlier is being calculated the actual value is used in the objective function rather than imputed value.

The rationale for the proposed model is that removing the observation from the analysis is not always the best approach as DEA is a relative measure of efficiency. Removing the observation will inherently affect all remaining DMUs and the DMU with the outlier will not have an efficiency score. The ability to correct for the unintended influence of outliers will provide a more accurate reflection of the relative efficiency and provide a more accurate projection onto the efficient frontier.
The definition of the proposed model is defined as follows:

$$\text{Max } \theta$$

Subject to:

$$\sum_{i=1}^{m} u_r[\,(\,y_{rj} * (1 - f_{rj}) + \,(a_{rj} * f_{rj})\,] \,\geq \theta y_{ro} \;\forall\, j \in \{1,..,n\}$$

$$\sum_{i=1}^{m} v_r[\,(x_{rj} * (1 - g_{rj}) + \,(b_{rj} * g_{rj})\,] \,\geq x_{ro} \;\forall\, j \in \{1,..,n\}$$

$$u_r \geq 0 \;\forall\, r \in \{1,..,s\}$$

$$v_i \geq 0 \;\forall\, i \in \{1,..,m\}$$

$$f_{rj} \in \{1,0\} \;\forall\, r \in \{1,..,s\}$$

$$g_{rj} \in \{1,0\} \;\forall\, r \in \{1,..,m\}$$

where:

f_{rj} and g_{rj} are flags for outliers in the outputs and inputs respectively.

a_{rj} and b_{rj} are alternative values for the outliers in the outputs and inputs respectively.

The flags for the output and inputs are predetermined binary variables resulting from an outlier analysis of the data. The alternative values for these outliers are median values for each output and input.

The common approach to identify outliers in a data set is to use the interval of the mean plus/minus a specified coefficient times the standard deviation. This coefficient is normally 2, 2.5 or 3 representing a poor conservative, mild conservative and very conservative respectively. According to the generally accepted "68-95-99.7" rule, by taking the mean plus three times the standard deviation one will include 99.73% of the observations. This means that only 0.27% of the data will be considered to be outliers.

Miller (1991) stated that there are three problems which arise using this approach. The first is that data is assumed to follow a normal distribution which includes the outliers. Secondly, the statistical measures used in the approach, which are the mean and the standard deviation, are heavily influenced by the outliers. The third problem with this approach was also stated by Cousineau and Chartier (2010) was that, this approach is not likely to be effective in smaller population.

Leys et al. (2013) proposed using the Median Absolute Deviation (MAD) as an alternative to the generally accepted approach of the mean with a three standard deviations buffer on the positive and negative side. Their approach states that the median plus/minus a specific coefficient times the MAD will provide a more robust measure for outlier detection. The MAD is defined in Fare et al. (2001):

$$MAD = b\, M_i\big(\big|x_i - M_j(x_j)\big|\big).$$

According to [36] the use of the MAD in the presence of outliers is one of the most effective and robust measures, however the final decision around using an appropriate exclusion criteria of outliers (specific coefficient of 2, 2.5 or 3) is subjective. Leys et al. (2013) suggested that 2.5 would be reasonable choice. This paper uses the approach proposed by Leys et al. (2013) to identify outliers in the data.

4. EFFICIENCY ANALYSIS ON MUNICIPALITIES

There have been numerous studies into the efficiency analysis of local municipalities across the globe spanning over three decades. Below is short summary of selected papers and the techniques that were applied. Table 4 and Table 5 provide a review of the literature on DEA and SFA studies respectively within the context of municipalities. Table 4 is an adaptation from Kutlar et al. (2012).

Table 4
Literature Review of DEA Studies

Authors	Methodology	Sample	Inputs, Output and Explanatory Variables
Deller and Nelson (1991)	DEA	446 municipalities in the US	This papers' variables included Number of full-time equivalent labour, together with road graders and the number of trucks. From a cost perspective, price of labour, capital and surfacing material was used with the cost of living index was used with the total surface material purchased.
Vanden Eeckaut et al. (1991)	DEA	235 Belgium municipalities	Total current expenditures Total population, proportion of people who are older than 65, number of people who live at the lowest life level, number of elementary school students, length of roads
De Borger, Kerstens, Moesen and Vanneste (1994)	FDH	589 Belgian municipalities	This study used the number of white and blue collar employees together with the capital stock, including the number of subsistence grants and students which are enrolled at schools. Public recreational area surfaces and the proportion of residents to non-residents where also included. Cost variables in the study include average personal income and grant. Educational factors taken into account included ratio of people with higher education.

Authors	Methodology	Sample	Inputs, Output and Explanatory Variables
Rouse et al. (1997)	DEA	62 New Zealand local governments	The various variables used in this study included the km of roads that have been refurbished as well as the general maintenance that has been undertaken. The road index in terms of roughness for both urban and rural roads where incorporated and the total index of the defects which exist were also used. The cost variables included total expenditure.
De Borger, and Kerstens, (1996b)	FDH	589 Belgian local governments	This study used similar variables to their earlier work in De Borger et al (1994), but included people over the age of 65, the rates and taxes levied by the municipalities, and the liberal/socialist ruling party in the form of a dummy variable. The total number of coalitions within the government were also taken into account.
Prieto and Zofio (2001)	DEA	209 Spain municipalities	Expected budget spending Potable water, waste, length of roads, number of units that illuminate the roads, cultural and sport background
Worthington and Dollery (2001)	DEA	103 New South Wales Municipalities	Inputs: Properties receiving DWMS, Occupancy rate, Population density, Population distribution, Cost of disposal index, collection expenditure. Outputs: Total garbage collected, Total recyclables collected, Implied recycling rate
Balaguer-Coll et al. (2002)	DEA	258 municipalities in Spain making using the panel data available	This study used cost variables such as total current expenditures and the number of illumination points. Other factors included the total population and the total tonnage of collected waste. The area of streets backgrounds, length of park areas, number of voters and the level of quality were also included.
Loikkanen and Susiluoto (2005)	DEA	353 Finland municipalities	Total current expenditures Daily child care houses, child care houses, central tooth care, older people’ home, handicapped home, school, number of libraries and their users

Authors	Methodology	Sample	Inputs, Output and Explanatory Variables
Afonso and Fernandes (2006)	DEA	51 municipalities in Lisbon	Cost variables included the general cost for admin and the spending per capita. Non-financial variables included social, educational and the amount of cultural services. The amount of waste and the performance of waste collectors were also taken into account.
Afonso and Fernandes (2008)	DEA	278 mainland Portuguese municipalities	Total municipal expenditure per inhabitant, Output variable include social services, together with the per capita for schools and the level of enrolments. The cultural services including water supply and the total amount of waste collection are used with territory organization and road infrastructure. The non-discretionary variables used in the study include purchasing power, the number of people with education at a secondary and tertiary level with their distance from their location to the district capital and finally the population density and variation.

Table 5 refers to the literature review conducted on studies which used SFA and the method to determine the relative efficiency of local municipalities.

Table 5
Literature Review of Stochastic Frontier Analysis Studies

Authors	Methodology	Sample	Inputs, Output and Explanatory Variables
Deller, Nelson and Walzer (1992)	Stochastic frontier	435 municipal areas in the US	The variables used in this study include the number of full-time labour and the road graders. The number of trucks and the surface material purchased for road maintenance. The cost variables included the price of labour and capital as well as the cost of the material used for resurfacing roads. The cost of living indexes was also used together with the total miles of gravel laid and the concentration of low and high bituminous roads.

Authors	Methodology	Sample	Inputs, Output and Explanatory Variables
Deller and Halstead (1994)	Stochastic frontier	104 municipalities in the US	The variables used in this study include the total road cost and the cost of the wages paid to labourers. The total cost for the graders and the number of trucks available. The cost of the capital and the miles of roads together with the education and training of the chief engineers.
De Borger, and Kerstens, (1996a)	DEA and FDH, including both deterministic and stochastic, frontiers analysis.	589 Belgian local governments	This study uses the same variables as stated in Table 4 by the same authors. This paper conducted earlier in the same year looked at the FDH and stochastics methods, whereas the paper published later in the year focused only on the FDH approach.
Athanass-opoulos and Triantis (1998)	DEA , stochastic frontier	172 Greece municipalities	The study uses total current expenditures as part of the cost variables and the number of settled families together with the average area and the length of spaces.
Wortinghton (2000)	DEA and stochastic method	166 Australia municipalities	Number of full time workers, financial expenditure Total population, number of equipment used to collect clean water and etc., length of rural and urban roads (km)
Boetti, Piancenza and Turati (2010)	DEA , SFA	262 Italian Municipalities	The inputs variables include the current expenditure for the use of general admin and the road maintenance costs. Service delivery variables include local mobility and the amount of waste collected together with the disposal of the waste. The education levels and the total services for elder people together with other social services are also included. The output variables include the number of inhabitants in the area and the length of roads with the municipality. Service delivery variables include the waste collected and the total school going children together with the number of people over age 75

The data used in the models are provided by Statistics South Africa. The variables relating to the number of people that have the services in the study were obtained from the Census 2011 data. These services included the number of households with:

- Access to piped water
- Access to flushing toilets
- Access to electricity for lighting.

From the Financial Census of Municipalities 2011 two values were obtained:

- The total expenditure

The descriptive statistics on the data Models are provided in the table below.

Table 6
Descriptive Statistics

	Variable	No. of people with access to Water	No. of people with access to Toilets	No. of people with access to Electricity	Total Expenses
District Municipalities	Mean	161 985	81 786	153 349	1 830 650
	Std. Dev.	95 952	61 648	95 905	1 266 595
	Minimum	18 966	11 256	17 046	303 487
	Maximum	452 949	247 740	422 460	4 450 560
Metropolitan Municipalities	Mean	757 110	635 854	686 883	17 309 817
	Std Dev.	446 147	410 106	411 078	10 907 561
	Minimum	217 932	145 182	180 915	3 677 488
	Maximum	1 415 004	1 282 011	1 303 044	32 046 907
Sample of Local Municipalities	Mean	28 995	13 851	27 855	308 917
	Std Dev.	25 542	17 649	25 481	367 837
	Minimum	1 668	444	1 326	19 919
	Maximum	113 922	98 538	126 045	2 001 525

The input for the model is the total expenditure and the outputs of the model are the total households which have access to: electricity, water and toilets. This model was applied to three sets of DMUs:

- The 44 District municipalities (DC)
- The eight metropolitan municipalities (M)
- Sample of 80 Local municipalities (L)

5. RESULTS

The preliminary results of the five conventional model (two deterministic and three stochastic) are shown below. The descriptive statistics of the various models indicate that the proposed model has a higher mean than that of the two deterministic models and more DMUs are found to be efficient. According to Bahari and Emrouznejad (2014), three questions were asked in the evaluation of the performance of the proposed method:

"1. How much the efficiency of removed DMU is changed?
2. How many DMUs are affected by the removed DMU?
3. How much is total change of the efficiencies?"

Since the proposed method in this paper does not remove observation but merely corrects for outlier this study poses the following questions to evaluate the performance of the new model:

1. How many DMUs had outliers?
2. How many DMUs in the reference set affected by the correction for outliers?
3. How drastic is the overall change of the efficiencies when compared to the conventional model?

Table 7 shows the descriptive statistics of the conventional models per sample.

Table 7
Results of the Model per Municipal Group

Variable	Type = DC					Type METRO					Type = SAMPLE				
	Mean	Std. Dev.	Range	N	No. Efficient	Mean	Std. Dev.	Range	N	No. Efficient	Mean	Std. Dev.	Range	N	No. Efficient
DEA-CCR	68.30%	16.31%	61.0%	44	4	86.63%	10.94%	32.0%	8	1	56.38%	19.19%	73.0%	80	5
DEA-BBC	77.11%	18.14%	60.0%	44	9	97.00%	5.95%	16.0%	8	6	62.08%	22.12%	70.0%	80	12
SFA-EXP	88.84%	2.97%	13.0%	44	.	84.63%	8.81%	25.0%	8	1	80.63%	7.27%	29.0%	80	-
SFA-TRUN	57.70%	13.81%	63.0%	44	1	87.25%	9.24%	26.0%	8	2	53.85%	10.17%	42.0%	80	-
SFA-HALF	84.07%	5.66%	25.0%	44	-	84.63%	8.81%	25.0%	8	1	79.24%	7.02%	28.0%	80	-

The results of the conventional models on the three samples show that there are significant differences in the means and standard deviations of the each approach. The deterministic models have higher standard deviations with varying mean values indicating that the efficiency score of the DMUs are fairly scattered across the range. The SFA-EXP and SFA-HALF models have much higher means and smaller standard deviations indicating that the efficiency score of these DMUs are clustered around the mean. This is the stochastic attempt to detecting random noise in the data. However this approach is not necessarily better.

Table 8 shows the results of the proposed model per municipal group. The overall results are quite similar to the parent model DEA-BCC. To fully understand the impact of the results of the new model it must be broken up into the DMUs with outliers and DMUs without outliers.

Table 8
Results of the Proposed Model per Municipal Group

Variable	Type = DC					Type METRO					Type = SAMPLE				
	Mean	Std. Dev.	Range	N	No. Efficient	Mean	Std. Dev.	Range	N	No. Efficient	Mean	Std. Dev.	Range	N	No. Efficient
New Model	77.7%	18.03%	60.0%	44	8	97.00%	5.95%	16.0%	8	6	65.14%	22.95%	70.0%	80	16

The comparisons of the proposed model to the conventional models are given in Table 9 below. In each case the movement of the efficiency score of the conventional models are given relative to the proposed model. Each comparison is broken up by the outlier flag indicating if that DMU had an outlier in one of its values. There are significant increase of efficiency scores to the DMUs without outliers and a decrease in the scores with outliers. Some DMUs with outliers also have an increase in the efficiency score, this can be attributed to the influence of the other DMUs with outliers.

In the DC sample, there were 4 DMUs with outliers. This leads to an increase of 281% affecting 35 DMUs when compared to the DEA-CCR model. This is evidence that the effect of the 4 DMUs with outliers had understated the efficiency scores of 35 DMUs. Additionally there were 4 DMUs without outliers that were found to be efficient in the new model that were not efficient in the DEA-CCR model.

The SFA-EXP and SFA-HALF models have overstated 30 and 28 DMUs, by 611% and 440% respectively. The SFA-TRUN model had understated the efficiency score and the new model showed 684% increase to the overall score affecting 39 DMUs.

The Metro sample had only 8 DMUs and no extreme values were found and the results of the new model are identical to its parent model DEA-BCC.

The Local sample had 80 DMUs, of which 7 were found to have an outlier. The DEA-CCR model understated 59 DMUs with a total of 509%. The DEA-BCC and SFA-TRUN models also understated the efficiency score by 193% and 769%, affecting 31 and 44 DMUs respectively. Similarly the SFA-EXP and SFA-HALF models overstated the efficiency score by 1436% and 1349% respectively. This culminated in an average reduction of 24% for the SFA-EXP model affecting 61 DMUs and 22% for the SFA-HALF model affecting 61 DMUs. The new model found 4 more efficient DMUs than the DEA-BCC model and 11 when compared to the DEA-CCR.

Table 9
Movement of the Conventional Model relative the New Model

			DEA-CCR			DEA-BCC			SFA-EXP			SFA-HALF			SFA-TURN		
Type	Change Relative to New Model	Outlier	Sum	Count	Average	Sum	Count	Average	Sum	Count	Average	Sum	Count	Average	Sum	Count	Average
DC	Down	No				-5%	3	-2%	-611%	30	-20%	-440%	28	-16%			
	Same	No	0%	5	0%	0%	36	0%							0%	1	0%
	Up	No	281%	35	8%	4%	1	4%	58%	10	6%	81%	12	7%	684%	39	18%
	Down	Yes				-1%	1	-1%									
	Same	Yes				0%	3	0%									
	Up	Yes	105%	4	26%				35%	4	9%	51%	4	13%	168%	4	42%
METRO	Down	No															
	Same	No	0%	3	0%	0%	8	0%	0%	1	0%	0%	1	0%	0%	2	0%
	Up	No	83%	5	17%				99%	7	14%	99%	7	14%	78%	6	13%
	Down	Yes															
	Same	Yes															
	Up	Yes															
LOCAL	Down	No							-1436%	61	-24%	-1349%	61	-22%	-119%	26	-5%
	Same	No	0%	14	0%	0%	42	0%							0%	3	0%
	Up	No	509%	59	9%	193%	31	6%	135%	12	11%	151%	12	13%	769%	44	17%
	Down	Yes							-9%	1	-9%	-8%	1	-8%			
	Same	Yes	0%	1	0%	0%	3	0%									
	Up	Yes	192%	6	32%	52%	4	13%	71%	6	12%	78%	6	13%	253%	7	36%

The proposed model decreased the efficiency score of these two SFA models in DMUs flagged as having no outliers. It can be argued that the SFA–EXP and SFA-HALF models have been significantly influenced by outliers resulting in a higher average score and smaller standard deviation. The comparison of the new model to both SFA-EXP and SFA-HALF models have corrected for the outlier influence reducing the efficiency.

6. CONCLUSIONS

DEA is a powerful technique in performance measurement and has various applications from banking to municipalities, hospital to libraries spanning over three decades. This technique is very sensitive to outliers due and can give inaccurate reflections of performance with the presence of outliers. There have been attempts to identify and remove observations with outliers but this comes with the cost of eliminating the entire DMU from the analysis. The removal of these DMUs will cause the results to be different to original sample due to the nature of the DEA model calculating the relative efficiency score and the removed DMU will not have an efficiency score at all. The proposed model in this paper outlines a method to accommodate for outliers and correct for the effects it can have on the relative efficiency score of DMUs with outliers whilst still keeping the outlier DMU in the analysis. The preliminary results have shown that when compared to the conventional methods (deterministic and stochastic) this proposed model has a significant increase in the efficiency score and a significant increase in the number of efficient DMU. The significance of not finding any outliers in the Metro sample, resulted in identical results to the base model DEA-BCC. This is an indication that the new model conforms to the conventional model when no outliers are present. This method can be used to accommodate for data with outliers and give a more accurate representation of the projection of inefficient DMUs to the efficient frontier.

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