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A SIMULATION STUDY OF SOME SAMPLE MEASURES OF SKEWNESS

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ABSTRACT

Holgersson (2010) has proposed a modified skewness measure "θ" that is based on the Pearson measure and the classical measure. He has claimed that the modified skewness measure uniquely determines the symmetry of variable for a wide range of distributions. However, he ignored in his study, the recently introduced measures. It is well known that the classical measure of skewness, $= \frac{\mu_3}{\sigma^3}$, measured by the standardized third moment is criticized in the literature. It is established that the Pearson measure does not portray skewness correctly in some situations and it does not preserve the ordering of skewness. The measure introduced by Holgersson (2010) inherits the drawbacks of both the classical and the Pearson measures of skewness. Doane and Seward (2011) have advocated the use of Pearson measure of skewness because it is simple to compute and interpret.

In the last two decades, a number of new measures of skewness have been introduced for a population data. However, sample versions of some measures of skewness are not discussed as their sampling distributions are not easy to obtain. Recently, Habib (2011) has obtained the sampling distribution of the sample version of the modified skewness measure "γ_T" introduced by Tajuddin (1999).

In this paper, we examine the performance of different measures of sample skewness by means of a simulation study and their applications on some real life data. We find that the sample version of the modified skewness measure "γ_T" introduced by Tajuddin (1999) performs at least better than the Pearson measure and it is simpler in computation and easier in interpretation than the Pearson measure of skewness. This measure performs better than classical measure and its modification given by Holgersson (2010), in the presence of outliers.

1. INTRODUCTION

The classical measure of skewness, "γ" $= \frac{\mu_3}{\sigma^3}$, measured by the standardized third moment is criticized in the literature [see, for example, Johnson and Kotz (1970) p. 253; Groeneveld and Meeden (1984); Xiaojun and Morris (1991)]. Groeneveld (1986) and Tajuddin (1999) have shown that the Pearson measure of skewness $\xi = \frac{\mu - m}{\sigma}$, where m is the median is not a reliable measure of skewness. Bowley (1920) has given the measure of skewness $B = \frac{Q_3 + Q_1 - 2\,Q_2}{Q_3 - Q_1}$, where $Q_i's, i = 1, 2, 3$ are the quartiles. Groeneveld and Meeden (1984) have discussed several extension of Bowley's measure of skewness and have introduced a measure of skewness $b_3 = \frac{\mu - m}{MAD\ (m)}$, where $MAD(m)$ is the mean absolute deviation about the median of a distribution. Arnold and Groeneveld (1995) have introduced a simple skewness measure of skewness, $\gamma_M = 1 - 2\,F(M)$, where M is the mode of a distribution. Tajuddin (1996) introduced another simple measure of skewness T in terms of conditional expectations and later, Tajuddin (1999) modified the measure T as $\gamma_T = 2\,F(\mu) - 1$. Brys et al. (2003) have introduced medcouple as a robust measure of skewness. Tajuddin (2012) has shown that medcouple is not a reliable measure of skewness. Habib (2011) has studied the sample version of the skewness measure "γ_T" introduced by Tajuddin (1999) and has discussed the sampling distribution of $\hat{\gamma}_T$. However, he compared the performance of "s" ($\hat{\gamma}_T$) with the Bowley's measure "B" and the sample version, "G", of the skewness measure "b_3" introduced by Groeneveld and Meeden (1984). Habib (2011, p.68) mentions that the measure s has overall lesser bias and the variance than those of other two measures considered in his study. Holgersson (2010) proposes the modified form of the classical measure of skewness. He has given a measure, "θ", that is based on the Pearson measure "ξ" and the standardized third moment, "γ".

Doane and Seward (2011) have recommended the use of Pearson measure of skewness, "$P = 3\xi$", over the classical measure simply because it is easy to compute and interpret. However, it is well known that the Pearson measure of skewness performs poorly (see, for example, MacGillivray, 1986; Groeneveld, 1986; Tajuddin, 1999, p.770; Tajuddin, 2012). Gibbons and Nichols (1979) have shown that the Pearson measure does not satisfies the skewness ordering of van Zwet (1964).The newly developed measures introduced by Tajuddin (1996, 1999) and Arnold and Groeneveld (1995) satisfy the skew ordering and are also easy to compute and interpret. On the other hand, the modified skewness measure introduced by Holgersson (2010) inherits the drawbacks of both the classical and the Pearson measures of skewness and it does not uniquely determines the symmetry of variable for a wide range of distributions.

A simulation study is carried on to compare the performance of different sample measures of skewness. In this study, we have considered the classical measure "γ", the Pearson measure ξ and the recently developed measures "b_3", "γ_T" and "θ" respectively introduced by Groenveld and Meeden (1984), Tajuddin (1999) and Holgersson (2010). We have ignored the measure "γ_M" introduced by Arnold and Groeneveld (1995) and the medcouple introduced by Brys et al. (2003) as these measures perform worse than the measure "γ_T". However, we have included the Pearson measure "ξ", due to the fact θ involves it.

Holgersson (2010) has proposed the measure:

$$\theta = \sigma^{-3}\, E\left(X - m\right)^3 = \gamma + \xi\,(3 + \xi^2) \qquad (1.1)$$

where, m is the median instead of the mean, γ is the classical measure of skewness and $\xi = (\mu - m)/\sigma$. He has demonstrated that θ is better than γ as a measure of skewness. However, in his study, he has not considered the recently introduced measures which are more efficient. In section 2, the computation of sample measures of skewness is described. This is followed by results obtained in section 3. In section 4, we discuss skewness values of different measures for some real life data. In section 5, conclusion of this study is presented.

2. SAMPLE SKEWNESS

Skewness of a sample is generally described by classical measure which can assume a very large value for a continuous population, however, whereas the sample skewness is bounded by the square root of the sample size [See Johnson and Lowe (1979)]. The purpose of this study is to compare the performance of measure introduced by Holgerssson (2010) with two competitive measures of skewness.

2.1 Some Sample Measures of Skewness

The following notation is used for different measures considered in this study. The sample version of θ given by (1.1) is taken as

$$\mathrm{H} = \frac{\sum (\mathrm{x_i} - \mathrm{median})^3}{n\hat{\sigma}^3}, \text{ where } \hat{\sigma}^2 = \frac{\sum (\mathrm{x_i} - \bar{\mathrm{x}})^2}{n}. \tag{2.1}$$

The sample version of the classical measure, designated by " C ", is taken as

$$\mathrm{C} = \frac{\sqrt{\mathrm{n}}\sum (\mathrm{x_i} - \bar{\mathrm{x}})^3}{(\sum (\mathrm{x_i} - \bar{\mathrm{x}})^2)^{1.5}} \tag{2.2}$$

The Pearson measure, denoted by P, is (mean-median)/s, where s^2 is the unbiased estimator of σ^2.

From (1.1), we can write

$$H = C + P(3 + P^2). \tag{2.3}$$

The sample version of the skewness measure " b_3 " given by Groeneveld (1986) is taken as

$$\mathrm{G} = \frac{n\,(mean - median)}{\sum |x_i - median|}.$$

The sample version of the skewness measure "γ_T" defined by Tajuddin (1999), is obtained after removing the median value from the sample and then considering

$$\mathrm{T} = \frac{number\ of\ observations\ less\ than\ median}{number\ of\ observations\ excluding\ the\ median} - 1.$$

2.2 Simulation Study

Minitab [See Ryan et al. 1985] is used to carry on the simulation study of the above measures. One thousand samples of size $n = 20$, 50 and 100 are obtained from the sampling distributions of Weibull, beta, gamma and log-normal distributions. For each sample, the above measures of skewness are computed and at the end average measures of skewness for 1000 samples values are obtained. The estimated values of the skewness measures are compared with the corresponding population measures. Results obtained are presented in the following section.

3. RESULTS

Weibull family is considered as an important family in reliability theory and it has been of interest in studying a skewness measure. Beta and Gamma families cover large range of distributions used in practical life. In addition, we have included log-normal distribution to study the skewness of samples.

3.1 Weibull Skewness

The family of Weibull distribution is described by

$$F(x) = 1 - \exp\ (-x^c), x > 0, c\ > 0.$$

The rth moment about origin is given by $\mu_r' = \Gamma\left(1 + \frac{r}{c}\right), r = 1, 2, 3.$

The median is $m = (\ln 2)^{\frac{1}{c}}$, and $E|x - m| = \mu\left[1 - 2G\left(\ln 2; \left(1 + \frac{1}{c}\right)\right)\right]$ where $G\ (x; a) = \int_0^x \frac{1}{\Gamma(\alpha)} t^{a-1} e^{-t} dt$. These results determine population skewness for different skewness measures. Table 1 presents the simulated skewness for different skewness measures for different sample sizes and different shape parameters of a Weibull distribution. For each value of c –the shape parameter, the true population skewness is shown as “True” in the table.

From Table 1, we have the following observations:

1. The average skewness values given by any skewness measure for $c \leq 2$ are no more than the true skewness values.
2. The estimated skewness for measures other than P, in general, increases and gets closer to the population value as the sample size increases. This behavior is not surprising as the chances are more for having large extreme values for larger samples yielding larger skewness for the samples. There is an exception with the Pearson measure.
3. The estimated skewness values for highly skewed distributions for the measures C and H for $c < 1$, increases exponentially with the decrease in the value of c. For $c = 0.1$, it is 69900. The estimated skewness C is bounded by $\sqrt{(n-1)}$, see Johnson and Lowe (1979).
4. The Pearson measure behaves very strangely; the value of skewness for the population measure as well as the sample measure decreases with the decrease in c for $c < 1$, whereas distributions get more skewed with the decrease in c. All other measures portray skewness more for smaller value of c. Van Zwet (1964) and others have shown that the measure P does not satisfy the requirements of a skewness measure.
5. The measure given by Holgersson is affected by the Pearson measure. “It can be seen from Table 1 that H is lesser than C for samples as well as for the population for Weibull (3.5), because P portrays negative skewness for a positively skewed distribution.”

Table 1
Average Skewness Values of Weibull (c) with different Sample Sizes

c	n	H	C	P	G	T
0.1	20	4.572	3.723	0.276	1.000	0.813
	50	6.661	6.097	0.186	1.000	0.884
	100	8.951	8.528	0.140	1.000	0.916
	True Value	69900	69900	0.002	1.000	0.978
0.2	20	4.419	3.383	0.333	0.987	0.719
	50	6.079	5.313	0.250	0.995	0.781
	100	7.790	7.166	0.205	0.997	0.807
	True Value	190.303	190.113	0.063	0.999	0.852
0.4	20	3.877	2.599	0.404	0.845	0.538
	50	4.975	3.831	0.365	0.882	0.574
	100	5.876	4.820	0.339	0.895	0.590
	True Value	12.215	11.353	0.280	0.905	0.603
0.5	20	3.571	2.307	0.400	0.747	0.464
	50	4.510	3.305	0.383	0.793	0.496
	100	5.133	3.972	0.370	0.802	0.505
	True Value	7.677	6.619	0.340	0.814	0.514
1	20	2.144	1.282	0.280	0.404	0.239
	50	2.542	1.625	0.297	0.427	0.256
	100	2.766	1.829	0.303	0.438	0.263
	True Value	2.949	2.000	0.307	0.443	0.264
2	20	0.766	0.471	0.098	0.130	0.082
	50	0.863	0.553	0.103	0.132	0.080
	100	0.938	0.595	0.114	0.145	0.089
	True Value	0.980	0.631	0.116	0.145	0.088
3.5	20	0.002	0.008	-0.002	-0.002	-0.003
	50	0.006	0.015	-0.003	-0.004	-0.001
	100	0.018	0.027	-0.003	-0.003	-0.003
	True Value	0.016	0.025	-0.003	-0.004	-0.002

3.2 Skewness of Beta Family

The following Beta family of distributions is considered. In two parameters Beta (α, β), first, we have taken $p = \alpha$ and $\beta = 1$.

$$f(x;\, p) = p x^{p-1} \quad 0 < x < 1,$$

We consider values of p as shown in Table 2. Average sample skewness values for $n = 100$ are given in Table 2 with population values given in parentheses.

Table 2
Skewness of Beta (p), $n = 100$

Shape Parameter p	Skewness Measures				
	H	**C**	**P**	**G**	**T**
0.1	4.127 (4.114)	2.686 (2.661)	0.450 (0.453)	0.987 (0.990)	0.574 (0.574)
0.2	3.401 (3.432)	1.670 (1.658)	0.528 (0.539)	0.830 (0.839)	0.397 (0.398)
0.4	1.988 (2.037)	0.861 (0.865)	0.360 (0.374)	0.453 (0.463)	0.208 (0.212)
0.5	1.459 (1.499)	0.633 (0.639)	0.269 (0.280)	0.326 (0.333)	0.151 (0.155)
0.8	0.425 (0.440)	0.194 (0.197)	0.077 (0.081)	0.090 (0.093)	0.044 (0.045)
1.0	-0.004 (0)	-0.004 (0)	0.000 (0)	0.000 (0)	0.000 (0)

From Table 2, we observe that skewness is an increasing function of $(p-1)$ for all measures other than the measure P.

We have also considered some unimodal distributions as given in Table 3. Here, we have considered different sample size as shown with true values in parentheses.

Table 3
Average Skewness of 1000 samples for Beta (α, β) distributions

(α, β)	*n*	*H*	*C*	*P*	*G*	*T*
(0.1, 10)	20	4.091	2.799	0.408	0.974	0.608
	50	4.937	3.776	0.370	0.988	0.632
	100	5.453	4.350	0.353	0.992	0.641
		(6.481)	(5.452)	(0.331)	(0.994)	(0.647)
(0.2, 10)	20	3.736	2.313	0.445	0.854	0.485
	50	4.327	2.929	0.438	0.886	0.501
	100	4.648	3.282	0.429	0.900	0.513
		(5.140)	(3.802)	(0.421)	(0.906)	(0.516)
(1, 10)	20	1.909	1.131	0.254	0.355	0.210
	50	2.212	1.340	0.283	0.390	0.229
	100	2.306	1.438	0.282	0.387	0.228
		(2.406)	(1.517)	(0.289)	(0.393)	(0.229)
(2, 10)	20	1.230	0.721	0.168	0.225	0.133
	50	1.700	0.841	0.279	0.229	0.136
	100	1.403	0.873	0.175	0.225	0.136
		(1.471)	(0.922)	(0.181)	(0.231)	(0.139)

From Table 3, we make the following observations:

1. The estimated skewness approaches the true value with the increase in sample size.
2. The measure P for $(\alpha, \beta) = (0.1, 10)$ shows lower skewness values for samples as well as the population skewness for lesser skewed distribution with $(\alpha, \beta) = (0.2, 10)$.

3.3 Skewness of Gamma Family

We have considered the one parameter gamma (α) given below.

$$f(x) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x}, x > 0, \alpha > 0.$$

We have considered $\alpha = 0.1$, 0.2, 0.5 which are highly skewed distributions, while $\alpha = 1.0$ which is the exponential distribution and $\alpha = 2$ is a moderately skewed distribution. Results are presented in Table 4.

Table 4
Skewness of Gamma (α), $n=100$

α	Skewness Measures				
	H	***C***	***P***	***G***	***T***
0.1	5.623 (7.298)	4.554 (6.325)	0.343 (0.314)	0.992 (0.995)	0.647 (0.655)
0.2	4.860 (5.739)	3.540 (4.472)	0.416 (0.401)	0.903 (0.912)	0.523 (0.529)
0.5	3.645 (4.040)	2.426 (2.828)	0.387 (0.385)	0.629 (0.636)	0.361 (0.365)
1.0	2.766 (2.950)	1.829 (2.000)	0.303 (0.307)	0.438 (0.443)	0.263 (0.264)
2.0	2.005 (2.107)	1.311 (1.414)	0.225 (0.227)	0.305 (0.306)	0.187 (0.188)

From Table 4, we note that the skewness values decrease as α increases for all measures excluding the Pearson's measure P. However, average sample values for measures H and C are lower than the true values, especially for $\alpha<1$.

3.4 Skewness of Log-Normal Family

We have considered the following log-normal distribution.

$$f(x) = \frac{1}{\sigma\, x\, \sqrt{2\pi}}\, exp\left\{\frac{(ln x)^2}{\sigma^2}\right\}, x > 0. \text{ We have considered } \sigma = 0.5, 1, 2, 3, 4.$$

Table 5
Skewness of Log-Normal (σ), $n=100$

σ	***H***	***C***	***P***	***G***	***T***
0.5	2.204 (2.421)	1.530 (1.750)	0.221 (0.220)	0.309 (0.307)	0.201 (0.197)
1	4.192 (7.112)	3.196 (6.185)	0.321 (0.300)	0.568 (0.576)	0.373 (0.383)
2	6.728 (414.715)	5.860 (414.359)	0.282 (0.118)	0.882 (0.906)	0.645 (0.683)
3	7.814 ($7.3*10^5$)	7.165 ($7.3*10^5$)	0.213 (0.011)	0.978 (0.992)	0.787 (0.866)
4	8.325 ($2.6*10^{10}$)	7.782 ($2.6*10^{10}$)	0.176 (0.000)	0.996 (1.000)	0.851 (0.955)

4. APPLICATIONS ON REAL LIFE DATA

We have included real life data considered by different authors who have studied some new measures of skewness.

4.1 Heat of Sublimation of Platinum Data

Habib (2011) considers a sample of 26 measurements of the heat of sublimation of platinum, given in Hampson and Walker (1961, p. 292). The data is displayed by a dot plot where, it can be seen that five observations are far from other values. We obtained skewness values for the data after omitting j largest values, j =0, 1, 2, 3, 4, 5. Results are presented in Table 6.

Table 6
Heats of Sublimation of Platinum Data

136.3	147.8	134.8	134.3	136.6	148.8	135.8	135.2	135.8
134.8	135.0	135.4	135.2	133.7	134.7	134.9	134.4	135.0
146.5	134.9	134.1	141.2	134.8	143.3	135.4	134.5	

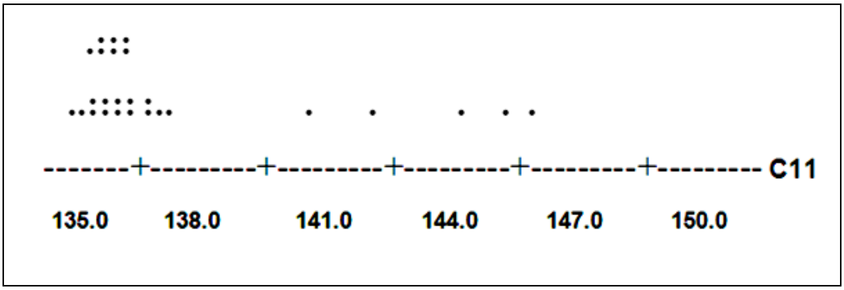

Fig. 1: Dot Plot of Platinum Data

Table 7
Skewness Values of Platinum Data

	$n = \mathbf{6}$	$n = \mathbf{25}$	$n = \mathbf{24}$	$n = \mathbf{23}$	$n = \mathbf{22}$	**21**
H	3.157	3.339	3.482	3.496	3.720	1.005
C	1.761	2.033	2.361	2.577	2.978	0.446
P	0.437	0.412	0.358	0.297	0.242	0.184
G	0.798	0.794	0.723	0.597	0.465	0.252
T	0.615	0.600	0.583	0.478	0.364	0.238

From Table 7, it is observed that both C -the classical measure and H –the measure due to Holgersson do not portray skewness of the data correctly; skewness of the data increases from $n = 26$ to $n = 22$ and abruptly decreases at $n = 21$ – observations with no outliers removed. The other three measures show decrease in skewness with the removal of more outliers.

4.2 Belgium CPI Data

Brys et al. (2003) have considered Belgium consumer price index (CPI) for monthly relative price differences of data consisting of 60 items. The data is presented in Table 8. It is positively skewed with some outliers on the right tail.

Table 8
Monthly Relative Price Differences of Belgium CPI Data (September 1978)

-0.036	0.328	2.216	0.129	-0.162	**8.903**	2.54	-0.316	-1.819	0.207
-0.778	-0.039	-0.181	0.048	-0.218	1.444	0.207	0.485	0.177	0.367
0.161	2.13	0.245	0.142	0.687	1.261	0.149	0.169	-0.049	0.129
0.091	0.024	-0.087	0.792	0.328	-0.132	0.014	0	1.943	0.311
-0.096	0.329	0.95	-0.077	-0.014	0	-0.294	0.071	0.007	1.089
0	2.664	0.038	0.109	0.018	0.099	-0.707	0	1.722	**8.414**

Two outliers highlighted are removed one at a time and skewness values obtained by different measures are shown in Table 9.

Table 9
Skewness of Belgium Data upon Removal of K Outliers

	$K = \mathbf{0}$	$K = \mathbf{1}$	$K = \mathbf{2}$
H	4.525	4.782	1.978
C	3.642	3.953	1.121
P	0.284	0.268	0.276
G	0.643	0.573	0.456
T	0.533	0.525	0.414

From Table 9, it is seen that on removal of first outlier, skewness shown by both C and H do not portray skewness of the data correctly. The other three measures show decrease in skewness with the removal of outliers.

4.3 Length of Stay data

Brys et al. (2003) obtain the skewness values for the length of stay for 500 patients with a maximum stay of 122 days and for the 495 patients who stayed up to 38 days. The data is shown in figure 2.

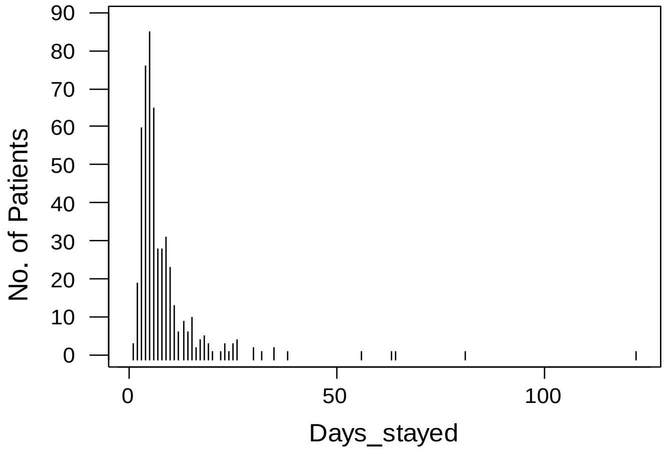

Fig. 2: Length of Stay Data of 500 Patients

We re-examine this data by considering number of patients ‘ N ’ who stayed 25 days or more for the length of stay data. In Table 10, we present the values of skewness measures and some other statistics for the length of stay for N patients who stayed up to certain number of days.

Table 10
Skewness of length of stay data with M- the Maximum stay (Days)

Measure	$M = 122$	$M = 38$	$M = 26$	$M = 25$
H	7.278	3.196	2.572	3.142
C	6.619	2.475	1.939	1.793
P	0.216	0.235	0.208	0.424
G	0.491	0.381	0.313	0.621
T	0.344	0.358	0.260	0.270
N	500	495	489	485
Mean	7.964	7.265	6.945	6.788
Median	6.000	6.000	6.000	5.000
StDev	9.098	5.371	4.543	4.217

From Table 10, we first note that the median length of stay, of N patients remains six days for $N = 489$ through $N = 500$ patients days and it drops down to five days for $N = 485$ patients. The value of G decreases gradually from 0.491 for patients who stayed up to 122 days to 0.313 patients who stayed up to 26 days. From the data, it is very clear that it gets lesser skewed as we delete some of the highest observations.

Interestingly, we note that the value of all the skewness measures which are based on the median show larger skewness values for the data of 485 patients with the length of stay up to 25 days than those of 489 patients with the length of stay up to 26 days. We display in figure 3, the data for 489 patients who stayed up to 26 days and in Figure 4, the data for 485 patients who stayed up to 25 days. From the results of Table 10 and figures 3 and 4, it is clear that the skewness measures based on median portray the skewness incorrectly. This happens because the median abruptly decreases from 6 to 5. Here, C is the only measure that portrays the skewness correctly for the data without outliers. The measure H is affected due to measure P. The measure G also gives a wrong message. However, it does portray the skewness of length of stay of 500, 495, 491, 490 and 489 patients, correctly.

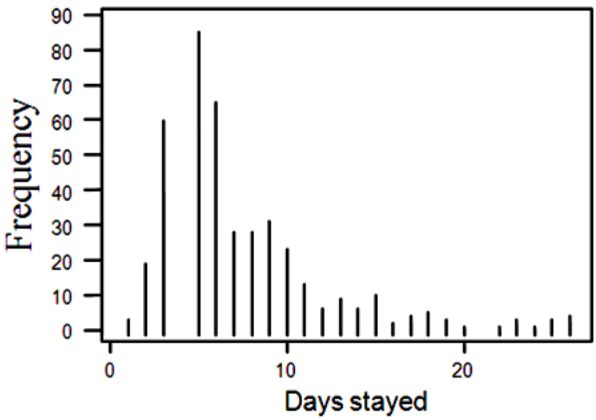

Fig. 3: Length of Stay Data of 489 Patients

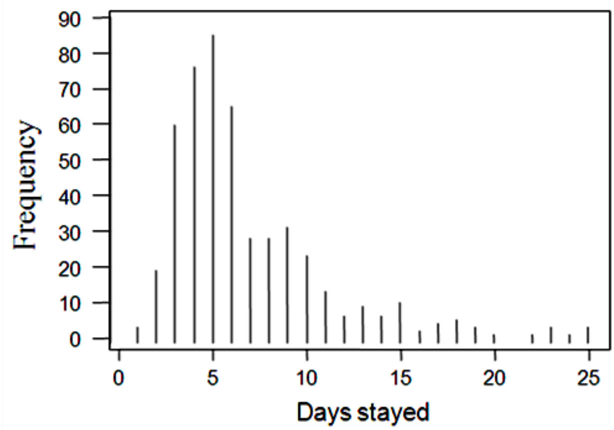

Fig. 4: Length of Stay Data of 485 Patients

5. CONCLUSION

1. The Pearson measure only maintains the skewness ordering in Table 7 for the platinum data. However it shows more skewness for lesser skewed distributions within families of Weibull, beta and lognormal distributions. [See Tables 1, 2, 3 and 5].
2. Skewness measure introduced by Holgersson (2010) inherits the drawbacks of both the classical and the Pearson measures of skewness. In Table 1, for $c = 3.5$, the estimated skewness as well as the true skewness portrayed by Pearson measure are negative, the consequence is that the Holgersson measure becomes smaller than the classical measure for the positively skewed distribution.
3. The estimated skewness value for highly skewed distributions for the measure C is bounded by $\sqrt{(n-1)}$, [see Johnson and Lowe (1979)] and hence the drawback is carried on by the measure H [See Table 1; estimated skewness for C and H for sample size 100 are respectively 8.528 and 8.951 while the true skewness for both is 69900]. In case of log-normal distribution with $\sigma = 4$ (Table 5), the sample skewness values for C and H, respectively, are 7.782 and 8.325 while population skewness is 2.649 x 10^{10}.
4. The Pearson measure performs well in the case of platinum data however, both C and H are affected badly by outliers and show higher skewness values after removing outliers from the right tail with positively skewed data, as is seen in Tables 7 and 9.
5. The measure G has a little effect due to outliers and it performs better than the other measures except in the case, where all measures based on median fail to portray the skewness of a data correctly [Table 10].
6. Almost all the real life data have shown clearly that H and C as well as P are unreliable measure of skewness.
7. In case of length of stay data, we observe that no measure of skewness may portray the skewness correctly in every situation. Therefore, we make the following recommendations.

In the absence of outliers, one may use the classical measure. In the presence of outliers, we recommend the use of T or G as measure of skewness.

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