Pak. J. Statist.
2015 Vol. 31(5), 557-574

IMAGE SEGMENTATION BASED ON FRACTIONAL NON-MARKOV POISSON STOCHASTIC PROCESS

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ABSTRACT

This paper introduces a novel region-based active contour model (ACM) with modified fractional non-Markov Poisson stochastic process (MFNMPS) to segment images within intensity inhomogeneity interface in a short time. Utilizing the global ACM, the technique integrates the MFNMPS in its edge enhancement. The MFNMPS with total energy is employed to enhance rapid contour movement within an enhanced of edge in the image texture. The fractional Poisson stochastic process with its counting function could classify the inhomogeneous object in a region in providing a smooth homogeneous region for segmentation with low computational cost. The MFNMPS with global energy allows the contour development to transfer toward the object founded on the preserved edges in providing improved segmentation. The conforming Euler-Lagrange is applied within the level set framework to minimize the energy. This study practices the proposed process to segment images with intensity inhomogeneity in segmenting several synthetic and medical images. With improved speed, the proposed method more accurately segments medical images compared with other baseline approaches.

KEYWORDS

Fractional calculus; level set method; active contour model; Poisson; local or global active contour; Non-Markov process.

1. INTRODUCTION

Medical images are known to have low quality due to two main factors: level of noise and level of intensity. Intensity inhomogeneity occurs in most medical images due to overlapping of information or pixels in the image [9,15,20]. Intensity inhomogeneity grounds a non-uniform distribution of intensity or inhomogeneous image texture in medical images [1,15,9,20]. Moreover, this sensation tips to difficulty in successfully segmenting medical images. The most common segmentation method comprises an active contour model (ACM) that uses a Gaussian filter with linear diffusion process to smooth the inhomogeneous image texture in medical images. Nevertheless, linear diffusion technique also eliminates important information regarding image details, such as the small edges at object boundaries, which may yield to boundary leakages [1,11,19]. Following the trails of calculated image pixel intensity or gradient in search of a complete object boundary [7,8,12,17,24,25], the basic ACM idea is to progressively change a contour or curve from its initial location in an image. Henceforth, the ACM fails to progress at the missing edge boundary. By contrast, the nonlinear diffusion technique steers the diffusion at apiece image point, thereby preserving every edge and maintaining the structure of image details [4,11,21].

It is known that the edge-based and region-based ACMs are the two classifications of ACMs. Due to the disadvantages of edge-based ACM which is sensitive to image noise and its constraint in medical image segmentation, many works focus to region-based ACMs. However, researchers discover the weaknesses of region-based ACMs which are sensitive to intensity inhomogeneity that creates and segment unwanted regions which outcomes in oversampling. This is due to the methods that highly utilized the global image properties. In both states, the object of interest in a medical image is not successfully segmented. Works in ACMs recently are focusing in utilizing both of edge-based and region-based ACMs strength that resulted in good achievement in medical image segmentation especially when dealing with interface of intensity inhomogeneity. However, the accuracy of the segmentation on medical images with severe intensity inhomogeneity remain unsolved.

Literature demonstrates a number of studies that utilize the nonlinear diffusion function in image segmentation [4,11,19,21,26] that lead to the use of generalization of nonlinear which is fractional calculus. The idea behind using the nonlinear diffusion function is to smooth an image texture and maintain its edge structure. Studies in ACM that use this function ongoing with the proposal of Perona and Malik. The Perona–Malik method offers the function's application in the anisotropic scale-space [19]. Their work purposes to reduce image noise without removing the parts of image content (i.e., edges, lines, and other details) significant for image interpretation. The smoothing procedure in the inhomogeneous object is applied in each iteration; thus, segmentation rapidity is slow with high computational cost. On the contrary, Ref. [23] depends on the nonlinear diffusion function together with the wavelet thresholding process to reduce image noise. The authors suggest a weighted diffusivity function that incorporates contextual discontinuities in the image. The diffusivity function is then applied on local image features to improve feature preservation capability along with noise removal. However, the technique concentrates on image de-noising without bearing in mind object segmentation in the image. Works on [10] demonstrate the use of fractional entropy descriptor in the segmentation of confocal microscopy images. The method aim in achieving the unsupervised region-based ACM in its contour in approaching the nuclei cells in microscopy images. However, the method may not achieving accurate segmentation on a severe intensity inhomogeneity images.

This paper introduces a novel region-based ACM with MFNMPS to improve segmentation process at low computational cost and to reduce the intensity inhomogeneity problem. The MFNMPS is responsible in solving problems that leads to an image which having random pixels with various intensity inhomogeneity that creates difficulty in segmentation. The fractional function inherits the nonlinear capability in enhancing the image structure and preserving important edges at the object boundary.

The nonlinear diffusion utilized in the MFNMPS function offers the most flexible contour-fitting functionality in the image thus provide an improved contour movement during its evolution [13,18]. The fractional Poisson stochastic process on the other hand, is able to count the inhomogeneous object found and classify them in a better way. Moreover, the strength of Poisson stochastic process with fractional function enable in solving problems with weak pixels found in the region which yields to missing edges. In the planned technique, this function is embedded within the global ACM energies to offer a stable contour evolution and reduce computational cost. This determination is skilled because the application of the function within the global energy ensures flexible contour movement, through an intensity inhomogeneity interface. This implies in decreasing the computational cost thus speed up the segmentation process. As the suggested technique works within the level set framework, the Euler- Lagrange is implemented to minimize the energy function. This is to make sure the level set contour/curve stop exactly on the object boundary in achieving improved segmentation.

The planned scheme has many rewards. First, the MFNMPS is applied in the contour to attain rapid movement with better contour movement effects. These effects speed up the segmentation process, thereby reducing the computational cost. Second, the suggested model smoothed the image texture while preserving its edges and enhancing the inhomogeneous object classification in the affected regions. Third, an amended segmentation in the intensity inhomogeneity interface is accomplished when the MFNMPS is implemented within the global and local ACMs and the energy is minimized based on the implementation of Euler- Lagrange.

Section 2 analyses the related ACM work that uses global properties in the segmentation process. Section 3 discusses the details of the anticipated technique. Section 4 clarifies the implementation framework. Section 5 documents the experimental setup. Section 6 provides the experimental results. Section 7 concludes the findings and outlines the future work.

2. RELATED WORK

Recently, works in ACM have been actively developed in order to improve the segmentation within interface of intensity inhomogeneity problem. In the earlier stages of image segmentation, synthetic images with clear boundary could easily be segmented by edge based methods such as geodesic active contour model. However, edge-based active contour model is sensitive to image noise and due to the initial contour which is dependent, segmentation on, medical images becomes difficult. Images that were affected with intensity inhomogeneity will also affects with noise depending on the noise level. Many methods of ACM develop an algorithm to achieve goal in classifying and merge the inhomogeneous object found in a regions to have region with homogeneous object. A well-known Chan-Vese method [14] which is region-based ACM, namely active contour model without edges assumed that region inside and outside the object must have homogeneous object. This means to achieve accurate segmentation the contour must meet condition of;

$$\inf_c \left\{F_1(C)+F_2(C)\right\} \approx 0 \approx F_1(C)+F_2(C) \tag{1}$$

where $F_1(C)$ and $F_2(C)$ is referring to the contour which is placed at the outside and inside region. As the method is based on minimizing energy within the level set method, the contour is placed on the contour when $\{F_1(C) \approx F_2(C)\} \approx 0$. The complete equation of C-V is given by;

$$E^{CV} = \lambda_1 \int_{inside(C)} |I(x) - c_1|^2 \, dx + \lambda_2 \int_{outside(C)} |I(x) - c_2|^2 \, dx \tag{2}$$

where c_1 and c_2 is constant and is based on linearity function. The c_1 and c_2 in the above equation are related to the global properties in the inside and outside regions which leads to sensibility to intensity inhomogeneity. As a result, medical images which were affected with intensity inhomogeneity segmentation on object of interest was not accurately segmented. To overcome the problem of C-V method, Chan-Vese developed an alternative method namely as piecewise-smooth (PS) model. The model aim at, making the intensity inside and outside regions as piecewise smooth function instead of constants. The algorithm for PS model is given by;

$$\begin{aligned}\varepsilon^{PS}(u^+, u^-, \phi) = &\int_\Omega |u^+ + I|^2 H(\phi) dx + \int_\Omega |u^- + I|^2 H(\phi) dx \\ &+ \mu \int_\Omega |\nabla u^+ + I|^2 H(\phi) dx + \int_\Omega |\nabla u^- + I|^2 H(\phi) dx + \nu \int_\Omega \nabla H(\phi) dx\end{aligned} \tag{3}$$

where μ and ν are fixed parameters and u^+ and u^- are the two smooth functions. In minimizing the energy the two smooth functions must be obtained by solving the partial differential equation foe each iterations. As a result, it leads to high computational cost.

Zhang et al. on the other hand, suggested a novel level set process that uses the Gaussian filter to regularize the selective binary LSF after each iteration [14]. The scheme is region-based, which lets for the selective implementation of either the local properties to segment only the exterior part or the global properties to segment both exterior and interior parts of an object in a given image. The LSF in this scheme is initialized to a constant, which has different signs inside and outside the contour [14]. The technique penalizes the zero level set to binary to escape the complex computation of re-initialization technique. The equation is formulated by

$$spf\left(I(x)\right) = \frac{I(x) - \frac{c_1 + c_2}{2}}{\max\left(\left|I(x) - \frac{c_1 + c_2}{2}\right|\right)} \tag{4}$$

where *spf* is the sign distance function proposed in the technique. This technique affords a large capture range and an anti-edge leakage capacity. Nonetheless, the contour is not accurately locked at the anticipated object boundary when a selective global ACM is utilized to segment both the exterior and interior parts of the medical images, mainly on images with severe intensity inhomogeneity.

3. MATERIALS AND METHODS

3.1 Proposed Design Method

The Gaussian filter is used to smooth the image texture for image segmentation and is normally applied consuming the linear diffusion function. This is normally implemented in previous ACM. The filter in the region-based contours is accountable for ending the contour movement at the correct object boundary. The Gaussian filter inclines to overlook neighboring pixels with less prominent gradients because it changes in straight lines. As an outcome, identification of the object boundary is less accurate. This work employs the MFNMPS, which is nonlinear to afford an efficient interruption procedure to classify and construct a new pixel or data points in an image. The goal is to provide a region with homogeneous objects. The procedure is best attained with contour or curve fitting to realize amended segmentation of the object to be segmented. Furthermore, the relationship of interruption procedure of MFNMPS method with Gaussian process will provide a non-straight rapid movement of contour within a given set or sampling of pixels in a region. Moreover, Poisson stochastic process will enable the counting of inhomogeneous object in a given region which will improve the segmentation process in a short time. A fractional non-Markov Poisson stochastic process has been developed based on fractional generalization of the Kolmogorov–Feller equation [26]. The fractional Kolmogorov–Feller equation, for probability distribution function P(x,y) is defined by;

$$\frac{\partial P(x,s)}{\partial s} = \int_{-\infty}^{\infty} [P(x-y,s) - P(x,s)\omega(y)dy], \quad P(x,0)\delta(x), \tag{5}$$

where here ω is probability density of the length y. Furthermore, randomness of step length is distributed in accordance with ω while s is the fractional distribution function of order α

$$\Psi(s) = \frac{\sin \pi\alpha}{\pi} \int_0^{\infty} \frac{e^{-\rho s} d\rho}{2\cos(\pi\mu) + \rho^{\alpha} + \rho^{-\alpha}}, \qquad 0 < \rho \leq 1, \tag{6}$$

which called a fractional Poissonian distribution. Let $P(x,s)$ be the probability of n items in the step or position s. The probability P satisfies the normalizing condition $\sum_{k=0}^{\infty} P(n,s) = 1$. The probability $P(\delta)$ can be formulated in term of teh probability distribution function

$$P(\delta) = \int_{\delta}^{\infty} \Psi(\acute{s})\ d\acute{s} = 1 - \int_0^{\delta} \Psi(\acute{s})\ d\acute{s}, \tag{7}$$

where

$$\int_0^{\delta} \Psi(\acute{s})\ ds' = \sum_{n=1}^{\infty} P(\bar{n},\delta) = 1 - e^{-\bar{n}\delta} \tag{8}$$

and

$$\Psi(\delta) = -\frac{d}{d\delta} P(\delta), \tag{9}$$

where $\bar{n}$ is the fractional mean. By combining Eqs. (7)–(9) , we finally obtain

$$\Psi(s) = \bar{n}e^{-\bar{n}\delta}. \tag{10}$$

The probability $P_{\alpha}(x,s)$ is expressed by the following special form of the fractional Kolmogorov–Feller equation

$$(\bar{n}, s) = \frac{s^{\alpha}\bar{n}}{n!}\sum_{k=0}^{\infty}\frac{(k+n)!}{k!}\frac{(-s^{\alpha}\bar{n})^{k}}{\Gamma(\alpha(k+n)+1)}. \tag{11}$$

The mean $\overline{n_\alpha}$ of the fractional Poisson process can be designed straightforwardly

$$\overline{n_\alpha} = \sum_{n=0}^{\infty} n\, P_\alpha(n, s) = \frac{\bar{n}\, s^{\alpha}}{\Gamma(\alpha+1)}. \tag{12}$$

In this method, we modify (11), by utilizing (12) to obtain the following modified fractional Poisson process

$$P_\alpha(\bar{n}_\alpha, s) = \frac{s^{\alpha}\overline{n_\alpha}}{n!}\sum_{k=0}^{\infty}\frac{(k+n)!}{k!}\frac{(-s^{\alpha}\overline{n_\alpha})^{k}}{\Gamma(\alpha(k+n)+1)}. \tag{13}$$

Consequently, by (13) the moment generation function $H_\alpha(\delta, s)$ can be defined by

$$\begin{aligned} H_\alpha(\delta, s) &= \sum_{n=0}^{\infty} e^{-\delta n}\ P_\alpha(\bar{n}_\alpha, s) \\ &= \sum_{n=0}^{\infty} \frac{1}{\Gamma(\alpha n+1)}\left[\overline{(n_\alpha}\, s^{\alpha}\ (e^{-\delta} - 1)\right]^{n}. \end{aligned} \tag{14}$$

For sufficient values of $0 < \alpha < 1$ and δ is large enough, we have that $H_\alpha(\delta, s) < 1$.

Poisson process is a function that arises regularly in describe numerous random phenomena including an area of closed, circular fluid motion rotating in the same direction (cyclone prediction), arrival times of calls to a call center in a hospital laboratory and call center, arrival times of aircraft to airspace around an airport and database transaction times.

Let C be a contour in an image Ω. The complete energy is defined as follows:

$$\begin{aligned} F\ (C, d_1, d_2) = {}& \lambda_1 \int_{in(C)} G(x) |\, I(y) - d_1(x)|^{\alpha}\ dxdy \\ &+ \lambda_2 \int_{out(C)} G(x) |\, I(y) - d_2(x)|^{\alpha}\ dxdy + \mu. Length(C) \end{aligned} \tag{15}$$

where λ_1 and λ_2 are two positive parameters, $G(x)$ is the Gaussian filter function in the proposed method and d_1, d_2 are defined by;

$$d_{1}(\phi) = \frac{\int_\Omega I(x).\mathrm{H}_\alpha(\phi)dx}{\int_\Omega \mathrm{H}_\alpha(\phi)dx}, \quad d_{2}(\phi) = \frac{\int_\Omega I(x).(1-\mathrm{H}_\alpha(\phi))dx}{\int_\Omega (1-\mathrm{H}_\alpha(\phi))dx} \tag{16}$$

where H_α is given in (14). Accordingly, d in Eq.(15) is not a constant value because it implies to a linear diffusion in a homogeneous environment. All locations in the image, including the edges, are equally smoothed [1,3,12,14,21] when a linear diffusion happens. This phenomenon cannot happen in medical images because every tiny image detail contains useful information. The level set method is implemented to solve the problem of topological changes in the ACM. Therefore, with the level set method, the new equation is given as follows:

$$\begin{aligned} F\ (\phi, d_1, d_2) = {}& \lambda_1 \int_{in(\phi)} G(x) |\, I(y) - d_2(x)|^{\alpha}\ H_\alpha(\phi) dxdy \\ &+ \lambda_2 \int_{out(\phi)} G(x) |\, I(y) - d_2(x)|^{\alpha}\ \big(1 - H_\alpha(\phi)\big) dxdy \\ &+ \mu. Length(\phi). \end{aligned} \tag{17}$$

As the proposed method works within the level set method (LSM) framework, the level set function (LSF) of LSM normally requires a contour placement to be frequently re-initialized to maintain the contour evolution stability but it require complex computations [6,16]. This study proposes an alternative technique by applying the Gaussian filter with MFNMPS in each contour movement. This technique provides a rapid and dynamic movement, which speeds up the segmentation process. Moreover, Gaussian filtering is used to enhance and preserve the edge, thereby maintaining the image structure. The edge is preserved toward the direction of the object boundary. The computational requirement for separating inhomogeneous objects within regions is simplified by applying the proposed in Gaussian filter with modify Poisson process. This is given by;

$$I(x) - (d_1 + d_2)^{\alpha} \tag{18}$$

where d_1 and d_2 are the two regions, and $I(x)$ is the original image. For a better segmentation process, the overall equation is applied to the image to improve the image intensity separation in each region. The power of α is the control parameter for improved segmentation outcome. If a large number is given to α, the contour will move further toward the segmented object boundary. If a small number is used, the contour will move nearer the segmented object. The choice of input for α depends on the distribution severity of the image intensity. This input desires to be properly tuned.

The contour did not stop at the exact object boundaries when only the global energy is utilized. To solve this problem, the energy function need to be minimized where the level set contour/curve must be on the object boundary. To realize this, we implement the distance measure based on the Euler- Lagrange technique. The equation for distance measure as stated in the third line of Eq.(17) is given by;

$$Length(\phi) = \mu L_{f\alpha}(\phi) \tag{19}$$

where $L_{f\alpha}(\phi)$ is the distance measure based on H_α function and is given by;

$$F_{\alpha}(\phi) = \int_{\Omega} |\nabla^{\alpha}\phi(x,y)|^{\alpha} H_{\alpha} dxdy \tag{20}$$

To minimize the energy function, $F_{\alpha}(\phi) = 0$ to make sure the contour placed exactly on the object boundary.

The algorithm for our proposed method can be summarized in the following steps:

1. The contour is initialized based on the curve evolution

   $$\mu = \begin{cases} d_1 & x \in \Omega_{in} \\ d_2 & x \in \Omega_{out} \end{cases}$$

   where d is not the constant used to implement the nonlinear diffusion concept.

2. d_1 and d_2 are computed based on Eq. (17).

3. The contour evolves based on the MFNMPS represented by H_α, $\alpha>0$.

4. The LSF is regularized and minimized within level set method.
5. The evolution of the LSF is checked for convergence. If the evolution does not converge, step number 2 is repeated.

In step 3, α is the control parameters adjusted to obtain better rapid contour movement toward the boundary of the object of interest. Giving a larger number of α parameters (i.e., more than 5) moves the contour far from the boundary. A too small input number (i.e., less than 1) draws the contour near or flat at the boundary of object of interest. The nonlinear function applied on the contour with the non-Markov Poisson that made it to exponential moves it forward and backward until the level set property is equal to $|\nabla\phi| = 1$. The fractional function parameter adjusted by the exponential variables work well when the local energy is efficiently adapted at the area with a high gradient level.

3.2 Implementation Framework

This work proposes a region-based ACM method that utilizes the global energies with MFNMPS function to improve the segmentation result on images with intensity inhomogeneity problem including medical images. MFNMPS function is applied to improve the contour evolution and enhance the image details especially the edges at the object boundary. The implementation framework of the proposed method in general is illustrated in Fig. 1.

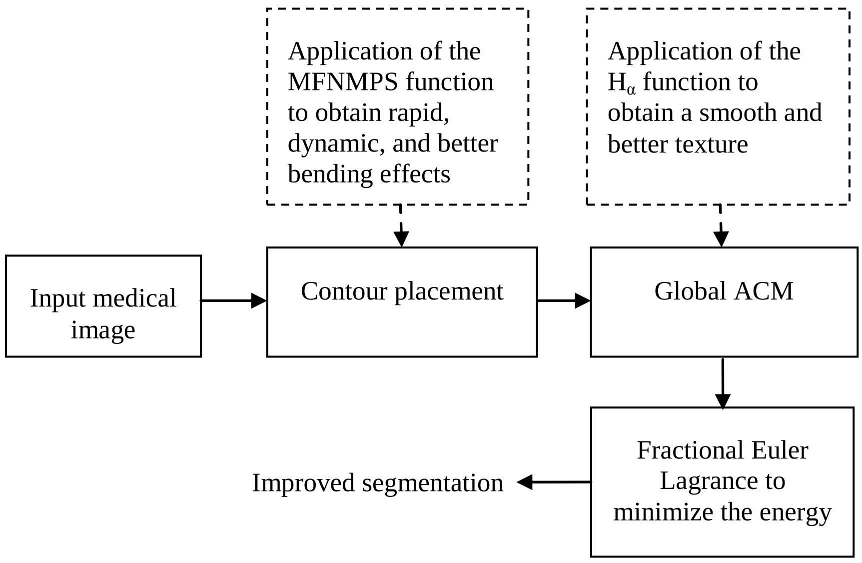

Fig. 1: Proposed Method Framework

MFNMPS function is embedded in the region-based ACM that utilized the global energies. The proposed method is aiming at having a better rapid and dynamically movement capability of the contour towards the object boundary. To make that happen, the Gaussian filter with implementation of fractional function is used to enhance and preserve the edges in the image structure, thereby maintaining the image details as fractional function is consuming the nonlinearity function. MFNMPS function on the other hand is having the ability to construct the inhomogeneous objects in regions into homogeneous object which enable the contour to fit in to get an improved segmentation of the object of interest. The application of controlled parameters (α) allows the contour to rapidly move forward or backward of the object of interest. If the controlled parameter values are large, the contour will move further to the boundary; otherwise, the contour may move nearer to the object boundary.

The distance measurement term utilizing the local energy is implemented along with the MFNMPS method to bring the contour near the object boundary. This works within the level set framework. During this process, the level set property is kept equal to $|\nabla\phi| = 1$. This value is used to stabilize and stop the contour movement when it reaches the object boundary. The value indicates that the contour is exactly on the boundary of the segmented object boundary. The contour stability during the evolution is maintained until the segmentation process is completed.

4. RESULTS AND DISCUSSION

In accordance to the implementation framework, we designed our experiment using Matlab R(2008b) on a 2.5 GHz with Intel Processor i5. The aim of these experiments is to evaluate the efficiency of the proposed method in segmenting the object of interest in synthetic and real images with the problem of intensity inhomogeneity. Besides, we evaluate the time taken by the proposed method in completing the segmentation process compared to other baseline methods.

Our experiment uses several synthetic images with unclear background which contains similarity in the gray scale levels. Later, we extended our experiments using several medical images which were affected with intensity inhomogeneity. To benchmark the performance of our proposed algorithm, we chose the following ACM methods that are commonly used in segmenting images, including medical images: Chan-Vese (C-V) and Selective Global and Local Active Contour Model (SGLACM) methods respectively. C-V method is a region-based ACM that endures high computational cost and produced over segmentation when dealing with intensity inhomogeneity. SGLACM manages to address the disadvantages of C-V method but does not provide stability of the contour during its evolution. In section 4.1, comparison is made among method of ACM and our method respectively on several synthetic images which later extended to medical images. The aim is to observe the performance and efficiency of the three methods including ours when applied to images with intensity inhomogeneity. From the result obtained we can observed the nature and performance of various ACM methods.

4.1 Comparisons with C-V and SGLACM Methods

In this section, demonstration on several synthetic and medical images are presented to evaluate and observed the proposed method when compared to some other baseline methods. First, several synthetic images are used as our experiments to evaluate the efficiency of the proposed method when compared to C-V and SGLACM methods. All synthetic images used in this section are having interface of intensity inhomogeneity and the background is having an unclear background. Fig.2 below illustrated an image of a car within an unclear background. Due to the effect of digital processing where the image was being enlarge until to the extend it creates many levels of gray scale which later difficult to be classified. As a result, the image of the car look unclear and the edge of the car's boundary is difficult to be trailed. Fig.2 illustrate in the first column is the segmentation outcome based on C-V method, the second column representing the segmentation outcome of SGLACM method and the last column is the segmentation outcome of our method. Due to various levels of gray scale, C-V method that is sensitive to intensity inhomogeneity problem could not successfully segment the image of car. At the background of the image that consists of various levels of intensity, the contour tends to create several regions which made the segmentation unsuccessful. The method on the other hand took about 250 iterations within 3.97s in completing the whole process of segmentation. In the second and last column depicts the segmentation results based on SGLACM and ours respectively. Both methods including ours managed to trail the objects' edges although it is not accurate. However, both methods are successful in avoiding the intensity inhomogeneity problem. The SGLACM method could segment the images in 55 iterations within 0.6s but our method managed to complete the segmentation process in 50 iterations within 0.53s. The successful of both methods may due to the smoothness of the image texture which allows the contour to rapidly move towards the object boundary. As for our method, the collaboration of the Poisson stochastic process with fractional function allows the interruption procedure in classifying the inhomogeneous objects to speed up the segmentation process.

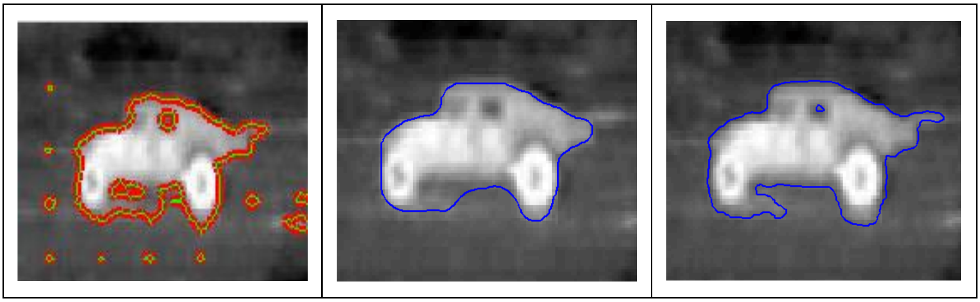

Fig. 2: Segmentation on synthetic image of a car. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method

To support the segmentation demonstration, we moved our experiment on image of a bean within a light background with intensity inhomogeneity problem. The nature of the bean image is similar to the nature of the car image but the background intensity is lighter than the background of the car image. However, the image texture of the bean shown in Fig.3 is not as smooth as image in Fig.2. Based on the experiment conducted, the first columns of Fig.3 depict the segmentation outcome based on C-V method. With 200 iterations in 5.88s the method segment the image with several other regions are segmented as well. This means due to its disadvantages which is sensitive to intensity inhomogeneity, the method did not managed to successfully segment the object of the bean. On the other hand, SGLACM method failed to detect the object in the image. This may due to the rough image texture that stop the contour from moving towards the object of a bean. However, our method managed to move the contour toward the object and managed to stop the contour on the object's edge. Moreover, the proposed method is robust to intensity inhomogeneity with cleaner outcome. The time taken to complete the segmentation for our method is within 40 iterations within 0.48s.

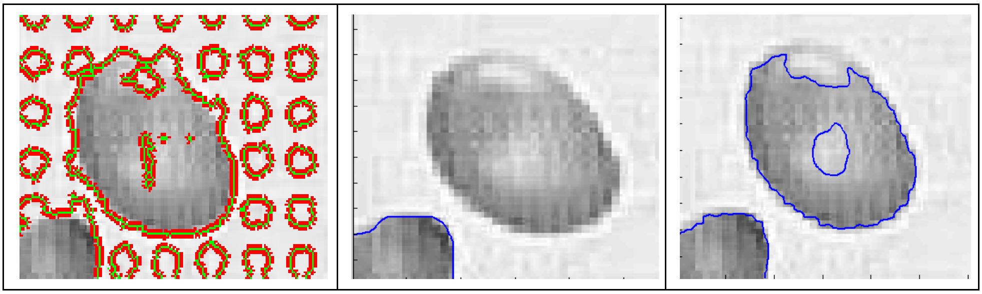

Fig. 3: Segmentation on synthetic image of a bean. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

Fig.4 and Fig.5 demonstrated an experiment conducted on two types of microscopic images representing the object of bacteria. The bacteria appeared in Fig.4 is having smaller sizes and scattered all over the image. The bacteria objects are placed within a smooth and solid gray scale level which is homogenous. On the other hand, image in Fig.5 is having bacteria whose sizes are big and they are placed nearer to each other. Those bacteria are placed within a rough background which having several layers of gray scale levels. The experiment conducted is to observe the performance of the proposed method in identifying and classifying the bacteria which are closed to each other. Based on the outcome, our proposed method shows excellent result where the bacteria objects on both images are well segmented and the outcome is clean. Image in Fig.4 is well segmented within 40 iterations in 1.5s and image in Fig.5 is successfully segmented within 40 iterations in 3.2s. The MFNMPS function applied in the proposed method managed to classify the inhomogeneous objects in a better way in the image and the fractional function which is nonlinear managed to provide a smooth image texture to allow the contour to move rapidly toward the object in the image.

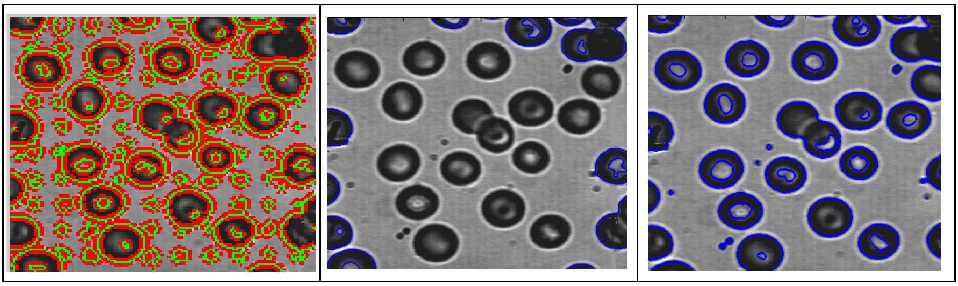

Fig. 4: Segmentation on microscopic image of bacteria. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

On the other hand, C-V and SGLACM method could not managed to successfully segment the image in both Fig.4 and Fig.5. C-V method segments the image of Fig.4 within 70 iterations in 10.88s and segment the image in Fig.5 within 240 iterations in12.82s. However, the outcome is not clean as many unwanted regions are segmented as well. Different for SGLACM method, due to both images are having intensity inhomogeneity, the method failed to segment both images accurately.

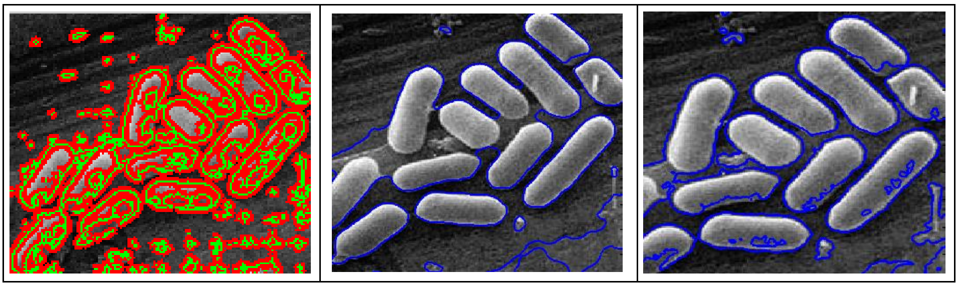

Fig.5: Segmentation on the second microscopic image of bacteria. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

Next, we continue our experiment on a challenging image which is image of cheetah skin with printed pattern. The image is chosen as the pattern on the cheetah skin is small and close to each other. This situation is challenging where the contour may ignored pixels with similar intensity level and may push the contour away from the object of interest. We would like to evaluate the efficiency of MFNMPS function applied with the proposed method on classifying the inhomogeneous object in regions that lead to improve segmentation. Fig.6 depicted the segmentation outcome. The first row shows the result based on C-V method. The method took about 250 iterations in 12.34s. However, the contour could not identify the small object scattered throughout the image as it provide many unwanted regions. The SGLACM on the other hand, could not manage to segment the image where the contour stop without segmenting objects in the center of the image. The proposed method however, managed to successfully segment each object found in the image in an excellent way. Within 220 iterations in 9.8s our proposed method efficiently segments the objects within the interface of intensity inhomogeneity.

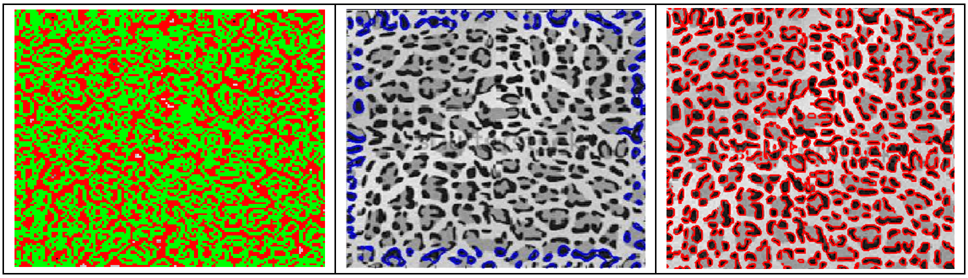

Fig.6: Segmentation on the second microscopic image of bacteria. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

In the experiments conducted in Fig.2 and Fig.3, we aim to demonstrate and proof that our proposed method managed to segment objects within an unclear background and consists of different levels of gray scale. The fractional function with Gaussian besides smoothing the image texture, provide a rapid movement of contour forward and backward towards the object boundary and giving a better bending effect at the corner of the object of interest. The nonlinear concept of fractional in the proposed method is exponential where it can be adjusted depending on the noise level of the image. This means the contour moved rapidly when it is far from the object boundary and moving slower when it reach near the object boundary. On the other hand, the experiments conducted from Fig.4 to Fig.6 illustrated on images which have many objects with various sizes and were placed close to each other. Other baseline methods either produce many unwanted regions or the contour having difficulty in moving toward the object to be segmented. For our method, according to Poisson stochastic process applied with fractional function, the technique managed to classify the inhomogeneous object and allow the contour to move surrounding the objects and improved the segmentation.

In our next experiment, medical images are used to demonstrate the efficiency of the proposed method in separating the object of interest within the intensity inhomogeneity interface. We provide three medical images with different modalities and they are MRI image of heart, X-Ray image of blood vessels and ultrasound image of cysts. The medical images presented having interface of intensity inhomogeneity and lots of noise that made it difficult for process of segmentation. The first medical image used in the demonstration is MRI image of heart. The object of heart having a strong boundary where the edge surrounding the heart is clear and do not have gaps or missing at edges. However, the internal part of the image that consist object of two hearts are having smooth image texture with boundary of unclear edges that was affected by intensity inhomogeneity. Fig.7 demonstrated the outcome of the segmentation based on C-V method in the first column, SGLACM in the second column and the proposed method at the last column. From the result obtained, among the three methods our method show cleaner outcome and produced improved segmentation. The contour with MFNMPS which uses the exponential regression enables the contour to move rapidly and bending effectively towards the object of heart. Moreover, the collaboration of fractional function with Poisson stochastic process managed to avoid the contour to segment any unwanted regions beside the object of interest. Our method had completed the segmentation with 40 iterations within 2.8s. The SGLACM method shows better segmentation outcome when compared to C-V method where the outcome is cleaner although the segmentation of heart is not complete. The method took 45 iterations in 1.8s. On the other hand, C-V method produced over segmentation in completing the segmentation process. The time taken to complete the segmentation is within 60 iterations in 4.5s.

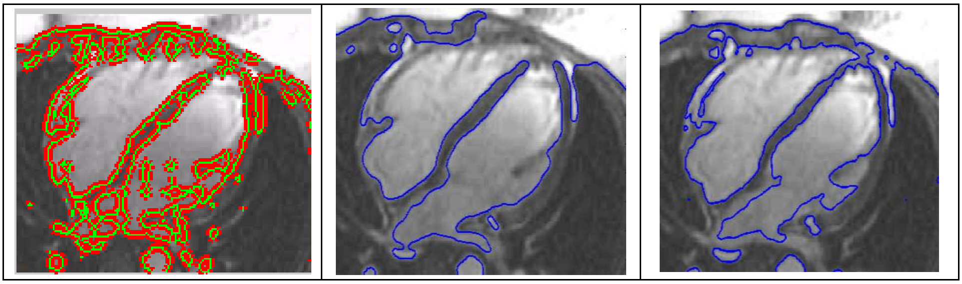

Fig.7: Segmentation on the second microscopic image of bacteria. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

The MRI image of heart was affected by intensity inhomogeneity mostly at the inner part of the image where the object of heart is placed. However, the image's background is dark and having less noise. Fig.8 illustrated the image of X-Ray of blood vessel. The image is having smooth texture where the object of vessel with intensity level which is similar to the background made it difficult to be seen. Moreover, the structure of blood vessel is long and thin that made it difficult to be accurately segmented. Due to this factor, the boundary of the blood vessel is difficult to be traced. We demonstrate the experiment to observe the effectiveness of the proposed method in segmenting the image. The first column of Fig.8 depicted the segmentation outcome based on C-V method, the second column is the result obtained from SGLACM and the last column is based on our method. Among the segmentation outcome our method and SGLACM method complete the segmentation of the blood vessel with less unwanted regions are segmented. However, in terms of accuracy at segmenting the exact boundary, our method could segment the blood vessel more accurate when compared to SGLACM method. Our method took only 40 iterations within 0.6s and SGLACM complete the segmentation within 60 iterations in 0.73s. On the other hand, C-V method produced over segmentation and complete the segmentation with 60 iterations in 6.28s.

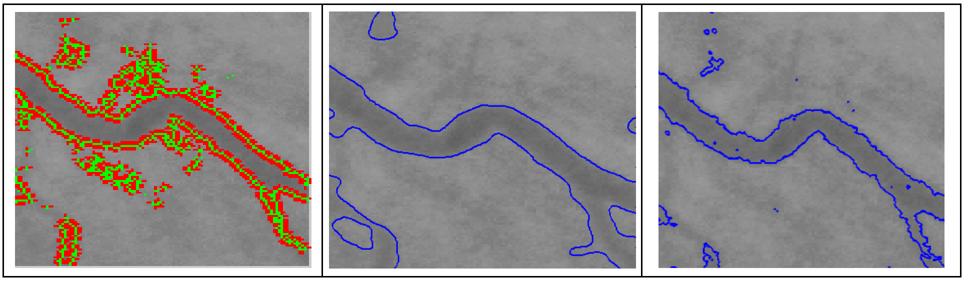

Fig.8: Segmentation on the second microscopic image of bacteria. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

Our last experiment depicted the medical image of cysts taken from the modality of ultrasound. Ultrasound image are known to have the worst quality in term of its noise, dark and rough texture. Due to this factor, interpretation of ultrasound images are difficult and can only be done based on prior knowledge. Fig.9 below presented the ultrasound image of cysts. The cyst is located in the center of the image with black intensity. The surrounding of the cysts is having a rough texture which leads to intensity inhomogeneity regions. Based on the three methods, experiments conducted on the image gave different outcome as shown in Fig.9. In the first row, show the outcome by C-V method that segment many unwanted regions beside the cyst. The method however, took 250 iterations in 8s to complete the whole process. The second column depicted the outcome by SGLACM method. Although the method complete the segmentation within 50 iterations in 1.31s but the cyst object is not segmented. The contour could not move and stop at regions contains many regions with inhomogeneous object. The last column depicted the segmentation outcome based on the proposed method. Our method managed to segment the cyst object and reduced the intensity inhomogeneity existed in the image. Our method took 40 iterations in 1.2s to complete the whole process.

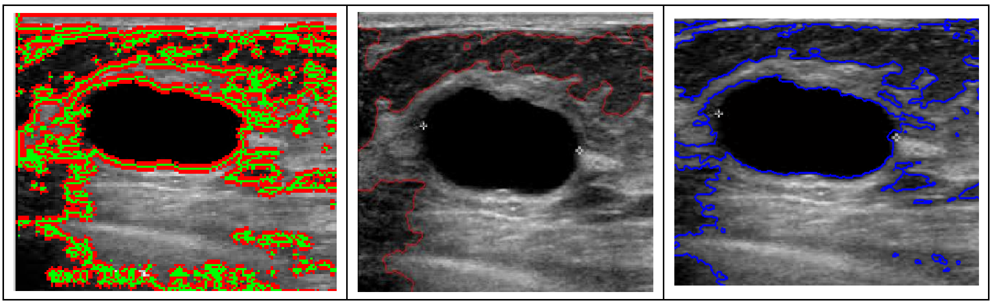

Fig.9: Segmentation on the second microscopic image of bacteria. The first column is the final result based on C-V method, the second column is based on Selective global method and the last column is based on our proposed method.

Demonstration conducted from Fig.7 to Fig.9 is based on three different modalities of medical images. The nature of medical images are different from synthetic images where the level of noise is high that leads to the problem of intensity inhomogeneity. Due to this, segmentation becomes difficult. We have demonstrated the experiment based on other methods of ACM including our method and have proof that our method show good potential when dealing with medical images with problem of intensity inhomogeneity. The outcome presented by our method show cleaner outcome which means the contour managed to move rapidly throughout the image and bending effectively at the corner of object of interest.

The fractional functional which is exponential since the non-Markov Poisson stochastic process is applied, allow adjustment of contour forward and backward toward the object boundary. The MFNMPS function applied with Gaussian filter managed to provide better image texture by enhancing the edges and reducing image noise. The sigma used in the proposed method is tuned based on the noise levels of the image. The sigma is adjusted to 3 when it is used to segment medical images in Fig.7 and Fig.8. The sigma is tuned to 1 when it is used to segment medical image in Fig.9. Besides improving the segmentation, our method managed to reduce the computational cost by speeding up the segmentation process. In the next section, we conclude our findings.

5. CONCLUSION

This paper presents a novel region-based ACM that used the modified fractional non-Markov Poisson stochastic process to improve the segmentation within the intensity inhomogeneity and speed up the contour evolution. The proposed fractional function with Gaussian filter provides improved image texture where the image details are enhanced while reducing the image noise. The fractional function which is nonlinear gave rapid movement capability of the contour. This paper proposes the application of MFNMPS to better classify the inhomogeneous object in regions in providing an improved image texture. A distance measurement based on fractional Euler Lagrange with local energy is implemented to accurately segment an object at its correct boundary within the level set framework. The energy function is minimized as the level set curve meet exactly on the object boundary. Either synthetic or medical images with different modalities, the proposed method provides improved segmentation of the object of interest at a lower computational cost than the other common ACM methods.

ACKNOWLEDGMENT

This research is supported by the Malaya University grant under the center of IPPP.

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