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A WEIGHTED MULTI-CRITERIA DISTRIBUTION MODEL FOR STUDENT ENROLLMENT INTO ACADEMIC PROGRAMMES

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ABSTRACT

A weighted multi-criteria model is built to maximize the distribution of students into academic programs of a department. The optimal distribution of students to these programs is determined by taking into account the constraints inherent in each academic program while complying with the limits of space capacity, the constraints of financial allocation, the number of instructors and affirmative action quotas. Each constraint has a weight attached to it that must be met. This model is applied to the School of Mathematical Sciences, Universiti Kebangsaan Malaysia, which consists of three academic programs. The successful application demonstrates the ability of the model to provide optimal distribution in compliance with the student intake requirement and constraints of each academic program in the department.

KEYWORDS

Affirmative action; allocation; constraints; goal programming; weights.

1. INTRODUCTION

Progress has been made in the field of mathematical programming and its use in academic environments. In order to emphasize the thrust of the academic institutions, academic administrators have to determine the types of courses being offered, the number of students to be enrolled, financial allocation and others that have to be dealt with every semester. In such an academic environment, administrators' decisions in course offerings and students enrolment are affected by the usual demand of certain programmes. However, the courses or programmes offered should indicate the thrust of the academic faculty, limited infrastructure, and the affirmative action requirement for government funded public universities.

Goal programming has been used extensively in many areas such as in management for Malaysian crops [Hassan et al. (2013a, 2013b, 2012a), Hassan and Sahrin (2012)], portfolio of Malaysian stock market [Hassan et al. (2012b)] and management of tourism activities [Hassan and Halim (2012)]. In the education field, goal programming has been used with AHP [Kiris (2014)] for course planning, applied to an e-learning system [Lin et al. (2014)], academic personnel management [Pal and Sen (2008)], resource allocation within academic units [Dharmapala and Saber (2007)], university admission policy [Elimam (1991)], optimal allocation of students to departments of an engineering college [Bafail and Moreb (1993)] and faculty-course-time assignment [Badri (1996)].

In this paper, a weighted goal programming model is developed which will optimize the distribution of students into academic programmes taking into account the expertise of academic staff, student capacity of each program, admission policies and financial allocations. This multi- objective model can be further refined to create a racial balance in each program based on the affirmative action policy and provide a fair distribution of student-to-faculty ratio. The goal programming model formulated will not take into account the selection of courses to be taught by certain instructors. Instead, weights will be used to apportion the students into academic programs in the department that will reflect the research thrust of the department. The weighted deviations are then included in the objective function to emphasise the ranking of goals.

2. MODEL DEVELOPMENT

Listed below are the input parameters, constraints and the objective function of the model in allocating students of a department, the School of Mathematical Sciences, to its three academic programmes of mathematics, statistics and actuarial science, for its three years undergraduate study.

Input Parameters

c_j = capacity of the number of first year students in programme j
r_j = student to faculty ratio required for programme j
q_j = minimum ratio of native students over the total students entering programme j
z_j = number of drop-out native students from programme j
t_j = total capacity of students in programme j proportionate to the no. of classes/halls
e_j = number of students enrolling into year two
h_i = number of students enrolling into year three

Variables

x_j = number of native students admitted into programme j
y_j = number of non-native students admitted into programme j
a_j = total number of first year students admitted into programme j
d_j = total number of students enrolled in programme j
f_i = total number of students enrolled in department i
l_j = number of faculty required for programme j
X = total number of first year native students admitted into the department
Y = total number of non-native students admitted into the department
A = total number of first year students admitted into the department

Constraints

The total number of students to be apportioned to M academic programs must be equal to the departmental intake of department i, where in our case would be the School of Mathematical Sciences with three programmes.

$$\sum_{j=1}^{M} x_j = X_i, \quad \sum_{j=1}^{M} y_j = Y_i, \quad \sum_{j=1}^{M} a_j = A_i, \quad \text{where } M = 3 \tag{2.1}$$

X_i , Y_i , A_i are 134, 88 and 222 respectively.

The total number of students accepted in programme j = sum of natives and non-natives accepted in programme j.

$$a_j = x_j + y_j \qquad j = 1,\dots, M$$

$$a_j - x_j - y_j = 0 \qquad j = 1,\dots, 3 \tag{2.2}$$

The total number of students accepted in a programme cannot exceed the capacity of the 1st year of that programme.

$$a_j \le c_j \qquad j = 1,\dots, M$$

$$a_j + d_{1j}^- - d_{1j}^+ = c_j\,, \quad j = 1,\dots, 3 \tag{2.3}$$

c_1, c_2 dan c_3 are 90, 80 and 70 respectively.

Note that underachievement and overachievement of the number of students accepted into each academic programme j, d_{1j}^- and d_{1j}^+, are allowed but are to be minimized. This will allow greater flexibility in allocating students in the programmes of a given department so as to be able to obtain a feasible solution. Since the number of programmes within a given department is limited, we do not foresee an infinitely many solutions.

The total number of enrollees cannot exceed the capacity of the programme from year 1 to year 3.

$$d_j \le t_j \qquad j = 1,\dots, M$$

$$d_j + d_{2j}^- - d_{2j}^+ = t_j, \qquad j = 1,\dots, 3 \tag{2.4}$$

t_1, t_2 dan t_3 are 260, 220 and 190 respectively.

Note that underachievement and overachievement of the total number of students enrolled into each academic programme j, d_{2j}^- and d_{2j}^+, are allowed but are to be minimized enabling a little room to allocate students in the programmes of a given department if the capacity is to be exceeded. We also do not foresee an infinitely many solution since the number of programmes within a given department is limited.

The total number of native students, less those who dropped out as a correction factor, must satisfy a certain given fraction of the total number of students accepted into the programme for affirmative action purposes.

$$x_j - z_j = q_j\, a_j \qquad j = 1,\dots, M$$

$$x_j - z_j - q_j\, a_j + d_{3j}^- - d_{3j}^+ = 0, \qquad j = 1,\dots, 3 \tag{2.5}$$

q_1, q_2 dan q_3 are 0.80, 0.49 and 0.48 respectively.

Balance of the number of students in each programme = admitted + enrolling into 2nd and 3rd year.

$$d_j = a_j + e_j + h_j \qquad j = 1,\dots, M$$

$$d_j - a_j = e_j + h_j \qquad j = 1,\dots, 3 \tag{2.6}$$

$e_j + h_j$ are 172, 134 and 121 for $j = 1,\dots, 3$

Size of students in department = sum of students in programmes.

$$f = \sum_{j=1}^{M} d_j$$

$$f = \sum_{j=1}^{3} d_j \quad \text{is } 649 \tag{2.7}$$

Student-faculty ratio x number of faculty members in programme j = total enrollees in programme j

$$r_j.l_j = d_j\,, \qquad j = 1,\ldots, M$$

$$r_j.l_j - d_j + d_{4j}^{-} - d_{4j}^{+} = 0, \qquad j = 1,\ldots, 3 \tag{2.8}$$

r_1, r_2 and r_3 are 14, 12 dan 26 respectively.

Note that under achievement and over achievement of the student-faculty ratio, d_{4j}^{-} and d_{4j}^{+}, are both allowed but are to be minimized. The presence of both deviational constraints will allow us to determine whether understaffing or overstaffing of faculty members in each academic programme has occurred. Corrective action in hiring an appropriate number of new teaching staff, sending some current staff for postgraduate studies or reassigning staff to another programme within the same department can thus be determined.

Bear in mind that the sum of student-faculty ratio of the mathematics and statistics programmes equals to that of the actuarial programme due to the first-year actuarial students enrollment into the core courses of the two former programmes.

Size of faculty members in department = sum of faculty members in M programmes.

$$l = \sum_{j=1}^{M} l_j$$

$$l - \sum_{j=1}^{3} l_j = 0 \tag{2.9}$$

For the budget estimation in programme j, the students cost in a programme does not vary much to other programmes within the same department since they share the same equipments and facilities. Hence the budget cost constraint is redundant and thus omitted.

Objective function

The criterion of optimization aims at maximizing the allocation of students accepted into the department with limitations imposed by programme capacity, number of academic staff, lecture halls or classrooms availability and affirmative action quota. Hence a priority system is devised to resolve conflicting constraints by

$$\max \quad \sum_{j=1}^{M} a_j = \sum_{j=1}^{M} x_j + y_j \quad \text{admission of first year students}$$

$$\max \quad f = \sum_{j=1}^{M} d_j \quad \text{number of enrollees in department} \tag{2.10}$$

$$\min \quad \sum_{j=1}^{M} x_j - q_j a_j \qquad \text{affirmative action quota, and} \tag{2.11}$$

$$\min \quad \sum_{j=1}^{M} l_j \qquad \text{number of faculty members} \tag{2.12}$$

Note that the objective function in this case, has to be rewritten as a single function of deviations and prioritized accordingly.

$$\text{Min Z} = \sum_{j=1}^{M} k_{1j}\left(d_{1j}^{-} + d_{1j}^{+}\right) + \sum_{j=1}^{M} k_{2j}\left(d_{2j}^{-} + d_{2j}^{+}\right)$$

$$+\sum_{j=1}^{M} k_{3j}\left(d_{3j}^{-} + d_{3j}^{+}\right) + \sum_{j=1}^{M} k_{4j}\left(d_{4j}^{-} + d_{4j}^{+}\right) \tag{2.13}$$

Note that the weights k_{ij} have values 1, 2 or 3 signifying the increasing rank where deviations are to be minimized with respect to the respective programmes. In our case for the first goal, admission requirement into the statistics programme has the highest rank $k_{12} = 3$, compared to those of mathematics $k_{11} = 2$, and actuary $k_{13} = 1$. The second goal of the capacity requirements of each programme is mathematics $k_{21} = 3$, statistics $k_{22} = 2$, and actuary $k_{23} = 1$, in decreasing order. However for the third goal, the affirmative action ratio for the statistics programme is highly ranked $k_{32} = 3$, compared to those of the actuary $k_{33} = 2$, and mathematics $k_{31} = 1$. The ranking of deviations of the fourth goal with respect to the number of faculty members in the three programmes is actuary $k_{43} = 3$, followed by mathematics $k_{41} = 2$, and statistics $k_{42} = 1$. Thus the values of the weights of deviations are

$$k_{11} = 2,\ k_{12} = 3,\ k_{13} = 1\ ;\quad k_{21} = 3,\ k_{22} = 2,\ k_{23} = 1\ ;$$

$$k_{31} = 1,\ k_{32} = 3,\ k_{33} = 2\ ;\quad k_{41} = 2,\ k_{42} = 1,\ k_{43} = 3. \tag{2.14}$$

3. ANALYSIS OF RESULTS

The output obtained with regard to the enrolment into three academic programmes of Mathematics, Statistics and Actuarial Science in the School of Mathematical Sciences is shown in Table 1.

Table 1
Results of the Weighted Model

Departmental programmes	Math	Stats	Act.Sc
Number of first year native students, *X*	65	39	30
Number of first year non-native students, *Y*	16	41	31
Number of first year students to be admitted, *A*	81	80	61
Number of academic staff in each programme, *L*	18	18	7
Number of students in each programme	253	214	182

Note that from the fourth column of the Table 1, the model suggest a mix of 30 native and 31 non-native students to be admitted into the actuary programme. The sum is less 9 than the admission capacity of 70 students. The values of the deviational variables with their priorities and respective weights are listed below. The objective value is found to be 74.24.

Weight = 3; $d_{12}^- = 0$, $d_{21}^- = 7$, $d_{32}^- = 0.2$, $d_{32}^+ = 0$, $d_{43}^- = 0$, $d_{43}^+ = 0$.
Weight = 2; $d_{11}^- = 9$, $d_{22}^- = 6$, $d_{33}^- = 0$, $d_{33}^+ = 0.72$, $d_{41}^- = 1$, $d_{41}^+ = 0$.
Weight = 1; $d_{13}^- = 9$, $d_{23}^- = 8$, $d_{31}^- = 0$, $d_{31}^+ = 0.2$, $d_{42}^- = 0$, $d_{42}^+ = 2$.

The results obtained comparatively mimicked the situational scenario, if not better. In fact the ratios

$X_1/A_1 = 0.802$ [0.805], $X_2/A_2 = 0.489$ [0.487], and $X_3/A_3 = 0.492$ [0.484]

provide a better affirmative action ratio to the target values of $Q_1 = 0.80$, $Q_2 = 0.49$ and $Q_3 = 0.48$ with highest weightage given to Q_2 compared to the others, if we are to compare with the current ratio given in the square brackets. The extra sum of staff in the department with regard to the L value allows the department to schedule study leave to its staff. Note that $L_3 = 7$ is equal to the current value. This signifies the conformity to the highest weightage given to the Actuarial Programme and a more flexible deviations to those of Mathematics and Statistics programmes. Note also the ratio

D_j/L_j = 14.06, 11.89, 26.00 [13.37, 11.78, 26.14] for j =1,2,3

and compare these values to the aspired student-staff ratio of

$R_1 = 14$, $R_2 = 12$, $R_3 = 26$.

Our model yields a better result in conforming to this ratio.

A weighted Mean Absolute Percentage Error (MAPE) analysis (Black, 2003) is established based on the error deviations of our model results and those of current values as indicated in Table 2. Type 1 is admission capacities into programmes. Type 2 is student capacity in programmes. Type 3 is affirmative action ratios while type 4 is student-staff ratios.

Table 2
Error Calculations for the Weighted Model

Type	**Weights w**	**Aspiration *X***	**Model**	$\lvert error \rvert$	**Current**	$\lvert error \rvert$
1	2	90	81	9	82	8
	3	80	80	0	78	2
	1	70	61	9	62	8
2	3	260	253	7	254	6
	2	220	214	6	212	8
	1	190	182	8	183	7
3	1	0.80	0.802	0.002	0.805	0.005
	3	0.49	0.489	0.001	0.487	0.003
	2	0.48	0.492	0.012	0.484	0.004
4	2	14	14.06	0.06	13.37	0.63
	1	12	11.89	0.11	11.78	0.22
	3	26	26.00	0	26.14	0.14

For our weighted model,

$$\text{The weighted MAPE} = \frac{\sum w_i \frac{|e_i|}{X_i} \times 100}{\sum w_i} = \frac{0.582351921}{24} \times 100 = 2.426\ \%.$$

Comparing this value to the MAPE of the current practice = 2.965 %, we note that the MAPE value for our model is less than that of the MAPE value of the current allocation. This signifies that our weighted allocation model gives a better result which is closer to the aspiration values compared to that of the current allocation practice.

4. CONCLUDING REMARKS

We have successfully obtained the results of the weighted goal programming and error analysis in the form of weighted Mean Absolute Percentage Error (MAPE) was conducted. It is shown that the MAPE value of our model is less than the MAPE value of the current practice. This shows that our model adhered closely to the requirements of the given aspiration levels. Based on the results obtained, we can verify that the results of the model conform to our requirement of fulfilling the goals in accordance to the corresponding weights. Hence the model can be used for future allocation of students to academic programmes of a department.

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