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ON ESTIMATION OF POPULATION MEAN IN THE PRESENCE OF MEASUREMENT ERROR AND NON-RESPONSE

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ABSTRACT

In this paper we study the problem of estimation of population mean in the presence of measurement error and non-response simultaneously using information on a single auxiliary variable. We have developed a new estimator of population mean and compared it with some existing estimators under the situations when measurement error and non-response occur simultaneously. The proposed estimators are theoretically compared with existing estimators. Empirical and simulation study is also conducted to assess the performance of proposed estimator.

KEYWORDS

Auxiliary variable, exponential estimator, bias, mean square error, measurement error, non-response.

1. INTRODUCTION

In sampling theory, there are two types of non-sampling errors: response error and non-response error. Response error or measurement error occurs when the reported value differs from the true value. Non-response error occurs when the researcher fails to collect information on one or more than one unit of the survey. Many estimators are available in literature without taking into account the response and non-response errors. Even if the non-sampling errors have been taken into account by the researchers, most of the available estimators are either for the case of measurement error or non-response separately.

In practice, the researcher often faces situations where some measurement error and non-response occur at the same time while collecting information. Measurement error may occur due to some over-reporting, under-reporting, memory failure by the respondents etc. while collecting data. The problem of non-response occurs when the researcher fails to collect information from some units in the survey due to a number of reasons like non-availability of the respondents at home, refusal to answer the questionnaire, lack of information etc.

The problem of estimation of population mean in the presence of measurement errors and non-response has been considered by many researchers. Hansen and Hurwitz (1946) studied the problem of non-response for the first time and suggested a procedure in which a sub-sample was drawn from the non-responding units in the sample and an extra effort was made for the collection of information from the non-respondents selected in the sub-sample. Cochran (1968) studied the effect of measurement errors on ordinary least squares estimates of regression coefficients and found that measurement errors may lead to inconsistent and biased estimates of the regression coefficients. Cochran (1977) suggested some new estimators of population mean using information on a single auxiliary variable for situations where some non-response occurs in the survey. Shalabh (1997) studied the classical ratio estimator of population mean in the presence of measurement errors. Manisha and Singh (2001) examined the effect of measurement errors on a new estimator which was a linear combination of the ratio and mean per unit estimators. Besides this, the problem of measurement error has also been studied by Fuller (1995), Manisha and Singh (2002), Srivastava and Shalabh (2001), Allen et al. (2003), Singh and Karpe (2008, 2009), Kumar et al. (2011) and Shukla et al. (2012) etc. The problem of non-response has also been studied by Rao (1986, 1987), Khare and Srivastava (1993, 1995, 1997, 2010), Tabasum and Khan (2004), Singh and Kumar (2008, 2011), Kumar and Sunil (2012, 2014) and Shabbir and Khan (2013) etc.

Many researchers have previously studied the problem of measurement error and non-response separately. In practice, it is possible that the problem of measurement error and non-response may occur at the same time. The objective of this paper is to suggest a new estimator of population mean using information of auxiliary variable in the presence of measurement error and non-response. A new estimator of population mean has been presented and a comparative study is made with the Hansen and Hurwitz (1946) estimator, Cochran’s (1977) estimator and Singh and Kumar (2008) estimator.

After introducing the concept of sampling, auxiliary information, nonresponse and measurement along with some important references, sample design in the case of measurement error and nonresponse is discussed in section 2 along with important notations. In section three, some existing estimators are discussed and section four contains the construction of new estimator along with the derivation of its mean square error. The proposed estimator is theoretically compared with existing estimators and efficiency conditions are deduced in section five. The empirical and simulation study is conducted in section six, the conclusion is provided in last section.

2. NOTATIONS

A simple random sample of size n is drawn from population of size N by simple random sampling without replacement (SRSWOR). It is assumed that the population is composed of two mutually exclusive groups, the N_1 respondents and the N_2 non-respondents, though their sizes are unknown. Let r_1 respond and r_2 do not respond. A sub-sample of size k ($k = r_2 / h$, $h \geq 0$) is taken from the non-respondents in the sample. Let (x_i^*, y_i^*) be the observed values and (X_i^*, Y_i^*) be the true values on two characteristics (x, y) respectively the ith (i = 1, 2,…, n) unit in the sample. Let the measurement errors be

$$u_i^* = y_i^* - Y_i^*, \qquad (2.1.1)$$

and

$$v_i^* = x_i^* - X_i^*, \qquad (2.1.2)$$

which are random in nature and are uncorrelated with mean zero and variances S_U^2 and S_V^2 respectively for the responding part of population. Let $S_{U(2)}^2$ and $S_{V(2)}^2$ be the variances associated with the measurement errors in study variable Y and auxiliary variable X respectively for the non-responding part of population. Let S_Y^2 and S_X^2 be the population variances of Y and X respectively for the responding part of population. Let $S_{Y(2)}^2$ and $S_{X(2)}^2$ be the population variances of Y and X respectively for the non-responding part of population. Let $\bar{Y}$ and $\bar{X}$ be the population means of Y and X respectively and let $\bar{y}$ and $\bar{x}$ be the sample means of Y and X respectively. Let ρ be the population correlation co-efficient between variable Y and X for the responding part of population and $\rho_{(2)}$ be the population correlation co-efficient between variable Y and X for the non-responding part of population. Let C_Y and C_X be the population co-efficient of variation for variable Y and X respectively for the responding part of population and $C_{Y(2)}$ and $C_{X(2)}$ be the population co-efficient of variation for variable Y and X respectively for the non-responding part of population.

We further assume that the mean of the study variable Y is unknown and the mean of auxiliary variable X is known.

3. SOME EXISTING ESTIMATORS

3.1 Hansen and Hurwitz (1946) Estimator

Hansen and Hurwitz (1946) suggested the following estimator when nonresponse occur

$$\bar{y}^* = \left(\frac{r_1}{n}\right)\bar{y}_{r1} + \left(\frac{r_2}{n}\right)\bar{y}_{k2}, \tag{3.1.1}$$

with variance

$$Var\left(\bar{y}^*\right) = \lambda S_Y^2 + \theta S_{Y(2)}^2, \tag{3.1.2}$$

where $\lambda = \frac{1}{n} - \frac{1}{N}$, $\theta = \frac{W_2\left(h-1\right)}{n}$, $W_2 = \frac{N_2}{N}$, $\bar{y}_{r1} = \frac{1}{r_1}\sum_{i=1}^{r_1} y_i$ and $\bar{y}_{k2} = \frac{1}{k}\sum_{i=1}^{k} y_i$.

If measurement error is taken into account, the variance of the Hansen and Hurwitz (1946) estimator can be written as:

$$Var^*\left(\bar{y}^*\right) = \lambda S_Y^2 + \theta S_{Y(2)}^2 + \lambda S_U^2 + \theta S_{U(2)}^2 . \tag{3.1.3}$$

3.2 Cochran's (1977) Estimator

Cochran (1977) proposed the following ratio-type estimator of population mean

$$t_C = \frac{\bar{y}^*}{\bar{x}^*} \bar{X} . \tag{3.2.1}$$

The mean square error of t_C in the presence of non-response and without measurement error is given as:

$$MSE(t_C) \approx \lambda \bar{Y}^2 \left(C_Y^2 + C_X^2 - 2\rho C_Y C_X \right) + \theta \bar{Y}^2 \left(C_{Y(2)}^2 + C_{X(2)}^2 - 2\rho_{(2)} C_{Y(2)} C_{X(2)} \right). \tag{3.2.2}$$

If measurement error is taken into account, the mean square error of t_C is given as:

$$MSE^*(t_C) \approx \lambda \bar{Y}^2 \left(C_Y^2 + C_X^2 - 2\rho C_Y C_X \right) + \theta \bar{Y}^2 \left(C_{Y(2)}^2 + C_{X(2)}^2 - 2\rho_{(2)} C_{Y(2)} C_{X(2)} \right)$$
$$+ \lambda \bar{Y}^2 \left[\frac{S_U^2}{\bar{Y}^2} + \frac{S_V^2}{\bar{X}^2} \right] + \theta \bar{Y}^2 \left[\frac{S_{U(2)}^2}{\bar{Y}^2} + \frac{S_{V(2)}^2}{\bar{X}^2} \right]. \tag{3.2.3}$$

3.3 Singh and Kumar (2008) Estimator

Singh and Kumar (2008) suggested the following chain-ratio-type estimator of population mean

$$t_{SK} = \bar{y}^* \left(\frac{\bar{X}}{\bar{x}^*} \right) \left(\frac{\bar{X}}{\bar{x}} \right). \tag{3.3.1}$$

The mean square error of t_{SK} in the presence of non-response and without measurement error is given as:

$$MSE(t_{SK}) \approx \lambda \bar{Y}^2 \left[C_Y^2 + 4C_X^2 - 4\rho C_Y C_X \right] + \theta \bar{Y}^2 \left[C_{Y(2)}^2 + C_{X(2)}^2 - 2\rho_{(2)} C_{Y(2)} C_{X(2)} \right]. \tag{3.3.2}$$

If measurement errors are taken into account, the mean square error of t_{SK} is given as:

$$MSE^*(t_{SK}) \approx \lambda \bar{Y}^2 \left[C_Y^2 + 4C_X^2 - 4\rho C_Y C_X \right] + \theta \bar{Y}^2 \left[C_{Y(2)}^2 + C_{X(2)}^2 - 2\rho_{(2)} C_{Y(2)} C_{X(2)} \right]$$
$$+ \lambda \bar{Y}^2 \left[\frac{S_U^2}{\bar{Y}^2} + 4 \frac{S_V^2}{\bar{X}^2} \right] + \theta \bar{Y}^2 \left[\frac{S_{U(2)}^2}{\bar{Y}^2} + \frac{S_{V(2)}^2}{\bar{X}^2} \right]. \tag{3.3.3}$$

4. PROPOSED ESTIMATOR

As in real life situation, at a time we can have such populations for those the correlation between study and auxiliary variables can be negative or positive. We require ratio estimator in the case of positive correlation and product estimator in the case of negative correlation. So there is need of such estimator that can handle both situations. In this connection we proposed the following estimator that combines the exponential ratio and exponential product estimator using probability weighting approach. The suggested estimator is

$$t_n = \bar{y}^*\left[\alpha \exp\left(\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right)+\left(1-\alpha\right)\exp\left(\frac{\bar{x}'^*-\bar{X}}{\bar{x}'^*+\bar{X}}\right)\right], \tag{4.1.1}$$

where α is a constant to be suitably chosen, and $\bar{x}'^* = \dfrac{N\bar{X}-n\bar{x}^*}{N-n}$.

In order to derive the mean square error of the proposed estimator, we introduce some further notations. Let

$$\omega_Y^* = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(Y_i^*-\bar{Y}\right), \tag{4.1.2}$$

$$\omega_U^* = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}U_i^*, \tag{4.1.3}$$

$$\omega_X^* = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(X_i^*-\bar{X}\right), \tag{4.1.4}$$

and

$$\omega_V^* = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}V_i^*. \tag{4.1.5}$$

Adding (4.1.3) and (4.1.4), we have

$$\omega_Y^* + \omega_U^* = \frac{1}{\sqrt{n}}\sum_{i=1}^{n}\left(Y_i^*-\bar{Y}\right)+\frac{1}{\sqrt{n}}\sum_{i=1}^{n}U_i^*.$$

On simplification, we get

$$\bar{y}^* = \bar{Y}+\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right). \tag{4.1.6}$$

Similarly from (4.1.4) and (4.1.5), we get

$$\bar{x}^* = \bar{X}+\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_V^*\right). \tag{4.1.7}$$

Further

$$\left.\begin{aligned}
E\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\right)^2 &= \lambda\left(S_Y^2+S_U^2\right)+\theta\left(S_{Y(2)}^2+S_{U(2)}^2\right)\\
E\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)^2 &= \lambda\left(S_X^2+S_V^2\right)+\theta\left(S_{X(2)}^2+S_{V(2)}^2\right)\\
E\left\{\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\right)\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)\right\} &= \lambda\rho S_Y S_X+\theta\rho_{(2)}S_{Y(2)}S_{X(2)}
\end{aligned}\right\} \tag{4.1.8}$$

Using (4.1.2) in (4.1.1), we get

$$t_n=\bar{y}^*\left[\alpha\exp\left(\frac{\bar{X}-\bar{x}^*}{\bar{X}+\bar{x}^*}\right)+(1-\alpha)\exp\left(\frac{\frac{N\bar{X}-n\bar{x}^*}{N-n}-\bar{X}}{\frac{N\bar{X}-n\bar{x}^*}{N-n}+\bar{X}}\right)\right]. \quad (4.1.9)$$

Using (4.1.6) and (4.1.7) in (4.1.9), we have

$$t_n=\left\{\bar{Y}+\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right)\right\}\left[\alpha\exp\left(\frac{\bar{X}-\left\{\bar{X}+\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_V^*\right)\right\}}{\bar{X}+\left\{\bar{X}+\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_V^*\right)\right\}}\right)\right.$$

$$\left.+(1-\alpha)\exp\left(\frac{\frac{1}{N-n}\left[N\bar{X}-n\left\{\bar{X}+\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_V^*\right)\right\}\right]-\bar{X}}{\frac{1}{N-n}\left[N\bar{X}-n\left\{\bar{X}+\frac{1}{\sqrt{n}}\left(\omega_X^*+\omega_V^*\right)\right\}\right]+\bar{X}}\right)\right].$$

Simplifying and ignoring terms of order greater than two, we have

$$t_n-\bar{Y}\approx\frac{1}{\sqrt{n}}\left(\omega_Y^*+\omega_U^*\right)-\frac{1}{2}\left\{\alpha\left(1-\frac{n}{N-n}\right)+\frac{n}{N-n}\right\}\frac{\bar{Y}}{\bar{X}}\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)$$

$$+\frac{1}{8}\frac{\bar{Y}}{\bar{X}}\left\{\alpha\left(\left(\frac{n}{N-n}\right)^2+3\right)-\left(\frac{n}{N-n}\right)^2\right\}\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)^2$$

$$-\frac{1}{2\bar{X}}\left\{\alpha\left(1-\frac{n}{N-n}\right)+\frac{n}{N-n}\right\}\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right). \quad (4.1.10)$$

Squaring both sides of (4.1.10) and taking expectation, we have

$$MSE^*\left(t_n\right)=E\left(t_n-\bar{Y}\right)^2\approx E\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\right)^2$$

$$+\frac{1}{4}\left\{\alpha\left(1-\frac{n}{N-n}\right)+\frac{n}{N-n}\right\}^2\frac{\bar{Y}^2}{\bar{X}^2}E\left(\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right)^2$$

$$-\left\{\alpha\left(1-\frac{n}{N-n}\right)+\frac{n}{N-n}\right\}\frac{\bar{Y}}{\bar{X}}E\left(\frac{\omega_Y^*+\omega_U^*}{\sqrt{n}}\frac{\omega_X^*+\omega_V^*}{\sqrt{n}}\right). \quad (4.1.11)$$

Using (4.1.8) in (4.1.11) and simplifying, we have

$$MSE^{*}\left(t_n\right) \approx \lambda \bar{Y}^2\left(C_Y^2+\frac{1}{4}\mu^2 C_X^2-\mu\rho C_Y C_X\right)$$
$$+\theta \bar{Y}^2\left(C_{Y(2)}^2+\frac{1}{4}\mu^2 C_{X(2)}^2-\mu\rho_{(2)} C_{Y(2)} C_{X(2)}\right)$$
$$+\lambda \bar{Y}^2\left(\frac{S_U^2}{\bar{Y}^2}+\frac{1}{4}\mu^2 \frac{S_V^2}{\bar{X}^2}\right)+\theta \bar{Y}^2\left(\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{1}{4}\mu^2 \frac{S_{V(2)}^2}{\bar{X}^2}\right), \qquad (4.1.12)$$

where $\mu=\left\{\alpha\left(1-\frac{n}{N-n}\right)+\frac{n}{N-n}\right\}$.

Differentiating (4.1.12) with respect to μ and equating to zero, the optimum value of μ is given by

$$\mu_{opt}=2\frac{\lambda\rho C_Y C_X+\theta\rho_{(2)} C_{Y(2)} C_{X(2)}}{\lambda\left(\frac{S_X^2+S_V^2}{\bar{X}^2}\right)+\theta\left(\frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2}\right)}=\mu_0 \text{ (say).} \qquad (4.1.13)$$

Using (4.1.13) in (4.1.12), the optimum mean square error of t_n is:

$$MSE_{opt}^{*}\left(t_n\right) \approx \lambda \bar{Y}^2\left(C_Y^2+\frac{1}{4}\mu_0^2 C_X^2-\mu_0\rho C_Y C_X\right)$$
$$+\theta \bar{Y}^2\left(C_{Y(2)}^2+\frac{1}{4}\mu_0^2 C_{X(2)}^2-\mu_0\rho_{(2)} C_{Y(2)} C_{X(2)}\right)$$
$$+\lambda \bar{Y}^2\left(\frac{S_U^2}{\bar{Y}^2}+\frac{1}{4}\mu_0^2 \frac{S_V^2}{\bar{X}^2}\right)+\theta \bar{Y}^2\left(\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{1}{4}\mu_0^2 \frac{S_{V(2)}^2}{\bar{X}^2}\right). \qquad (4.1.14)$$

5. EFFICIENCY COMPARISON

5.1 Proposed Estimator vs. Hansen and Hurwitz (1946) Estimator

The proposed estimator is more efficient than the Hansen and Hurwitz (1946) estimator if

$$Var^{*}\left(\bar{y}^{*}\right)-MSE^{*}\left(t_n\right) \geq 0 \cdot \qquad (5.1.1)$$

Using (3.1.5) and (4.1.14) in (5.1.1), we have

$$\lambda S_Y^2+\theta S_{Y(2)}^2+\lambda S_U^2+\theta S_{U(2)}^2-\lambda \bar{Y}^2\left(C_Y^2+\frac{1}{4}\mu^2 C_X^2-\mu\rho C_Y C_X\right)$$
$$-\theta \bar{Y}^2\left(C_{Y(2)}^2+\frac{1}{4}\mu^2 C_{X(2)}^2-\mu\rho_{(2)} C_{Y(2)} C_{X(2)}\right)$$
$$-\lambda \bar{Y}^2\left(\frac{S_U^2}{\bar{Y}^2}+\frac{1}{4}\mu^2 \frac{S_V^2}{\bar{X}^2}\right)-\theta \bar{Y}^2\left(\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{1}{4}\mu^2 \frac{S_{V(2)}^2}{\bar{X}^2}\right) \geq 0,$$

or if

$$\lambda\bar{Y}^2\left[\frac{1}{4}\mu^2\frac{S_X^2+S_V^2}{\bar{X}^2}-\mu\rho C_Y C_X\right]+\theta\bar{Y}^2\left[\frac{1}{4}\mu^2\frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2}-\mu\rho_{(2)}C_{Y(2)}C_{X(2)}\right]\le 0. \tag{5.1.2}$$

(5.1.2) is true if

$$\frac{1}{4}\mu^2\frac{S_X^2+S_V^2}{\bar{X}^2}-\mu\rho C_Y C_X\le 0, \tag{5.1.3a}$$

and

$$\frac{1}{4}\mu^2\frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2}-\mu\rho_{(2)}C_{Y(2)}C_{X(2)}\le 0. \tag{5.1.3b}$$

From (5.1.3a), we have

$$\rho\ge\frac{1}{4}\mu\frac{C_X}{C_Y}\left(1+\frac{S_V^2}{S_X^2}\right), \tag{5.1.4a}$$

and

$$\rho_{(2)}\ge\frac{1}{4}\mu\frac{C_{X(2)}}{C_{Y(2)}}\left(1+\frac{S_{V(2)}^2}{S_{X(2)}^2}\right). \tag{5.1.4b}$$

5.2 Proposed Estimator vs. Cochran's (1977) Estimator

The proposed estimator is more efficient than Cochran's (1980) estimator if

$$MSE^*(t_C)-MSE^*(t_n)\ge 0. \tag{5.2.1}$$

Using (3.2.3) and (4.1.13) in (5.2.1), we have

$$\begin{aligned}&\lambda\bar{Y}^2\left(C_Y^2+C_X^2-2\rho C_Y C_X\right)+\theta\bar{Y}^2\left(C_{Y(2)}^2+C_{X(2)}^2-2\rho_{(2)}C_{Y(2)}C_{X(2)}\right)\\&+\lambda\bar{Y}^2\left[\frac{S_U^2}{\bar{Y}^2}+\frac{S_V^2}{\bar{X}^2}\right]+\theta\bar{Y}^2\left[\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{S_{V(2)}^2}{\bar{X}^2}\right]-\lambda\bar{Y}^2\left(C_Y^2+\frac{1}{4}\mu^2C_X^2-\mu\rho C_Y C_X\right)\\&-\theta\bar{Y}^2\left(C_{Y(2)}^2+\frac{1}{4}\mu^2C_{X(2)}^2-\mu\rho_{(2)}C_{Y(2)}C_{X(2)}\right)\\&-\lambda\bar{Y}^2\left(\frac{S_U^2}{\bar{Y}^2}+\frac{1}{4}\mu^2\frac{S_V^2}{\bar{X}^2}\right)-\theta\bar{Y}^2\left(\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{1}{4}\mu^2\frac{S_{V(2)}^2}{\bar{X}^2}\right)\ge 0,\end{aligned}$$

or if

$$\lambda\bar{Y}^2\left[\left\{1-\frac{1}{4}\mu^2\right\}\frac{S_X^2+S_V^2}{\bar{X}^2}-2\rho C_Y C_X\left\{1-\frac{1}{2}\mu\right\}\right]$$

$$+\theta\bar{Y}^2\left[\left\{1-\frac{1}{4}\mu^2\right\}\frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2}-2\rho_{(2)}C_{Y(2)}C_{X(2)}\left\{1-\frac{1}{2}\mu\right\}\right]\geq 0. \quad (5.2.2)$$

(5.2.2) is true if

$$\left\{1-\frac{1}{4}\mu^2\right\}\frac{S_X^2+S_V^2}{\bar{X}^2}-2\rho C_Y C_X\left\{1-\frac{1}{2}\mu\right\}\geq 0, \quad (5.2.3a)$$

and

$$\left\{1-\frac{1}{4}\mu^2\right\}\frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2}-2\rho_{(2)}C_{Y(2)}C_{X(2)}\left\{1-\frac{1}{2}\mu\right\}\geq 0. \quad (5.2.3b)$$

From (5.2.3a), we have

$$\rho\leq\frac{1}{2}\left\{1+\frac{1}{2}\mu\right\}\frac{C_X}{C_Y}\left(1+\frac{S_V^2}{S_X^2}\right), \quad (5.2.4a)$$

and from (5.2.3b), we have

$$\rho_{(2)}\leq\frac{1}{2}\left\{1+\frac{1}{2}\mu\right\}\frac{C_{X(2)}}{C_{Y(2)}}\left(1+\frac{S_{V(2)}^2}{S_{X(2)}^2}\right). \quad (5.2.4b)$$

5.3 Proposed Estimator vs. Singh and Kumar (2008) Estimator

The proposed estimator is more efficient than Singh and Kumar (2008) estimator if

$$MSE^*\left(t_{SK}\right)-MSE^*\left(t_n\right)\geq 0. \quad (5.3.1)$$

Using (3.3.3) and (4.1.13) in (5.3.1), we have

$$\lambda\bar{Y}^2\left[C_Y^2+4C_X^2-4\rho C_Y C_X\right]+\theta\bar{Y}^2\left[C_{Y(2)}^2+C_{X(2)}^2-2\rho_{(2)}C_{Y(2)}C_{X(2)}\right]$$

$$+\lambda\bar{Y}^2\left[\frac{S_U^2}{\bar{Y}^2}+4\frac{S_V^2}{\bar{X}^2}\right]+\theta\bar{Y}^2\left[\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{S_{V(2)}^2}{\bar{X}^2}\right]-\lambda\bar{Y}^2\left(C_Y^2+\frac{1}{4}\mu^2C_X^2-\mu\rho C_Y C_X\right)$$

$$-\theta\bar{Y}^2\left(C_{Y(2)}^2+\frac{1}{4}\mu^2C_{X(2)}^2-\mu\rho_{(2)}C_{Y(2)}C_{X(2)}\right)-\lambda\bar{Y}^2\left(\frac{S_U^2}{\bar{Y}^2}+\frac{1}{4}\mu^2\frac{S_V^2}{\bar{X}^2}\right)$$

$$-\theta\bar{Y}^2\left(\frac{S_{U(2)}^2}{\bar{Y}^2}+\frac{1}{4}\mu^2\frac{S_{V(2)}^2}{\bar{X}^2}\right)\geq 0,$$

or if

$$\lambda \bar{Y}^2 \left[4\left\{1-\frac{1}{16}\mu^2\right\} \frac{S_X^2+S_V^2}{\bar{X}^2} - 4\rho C_Y C_X \left\{1-\frac{1}{4}\mu\right\}\right]$$

$$+\theta \bar{Y}^2 \left[\left\{1-\frac{1}{4}\mu^2\right\} \frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2} - 2\rho_{(2)} C_{Y(2)} C_{X(2)} \left\{1-\frac{1}{2}\mu\right\}\right] \geq 0. \quad (5.3.2)$$

(5.3.2) is true if

$$4\left\{1-\frac{1}{16}\mu^2\right\} \frac{S_X^2+S_V^2}{\bar{X}^2} - 4\rho C_Y C_X \left\{1-\frac{1}{4}\mu\right\} \geq 0, \quad (5.3.3a)$$

and

$$\left\{1-\frac{1}{4}\mu^2\right\} \frac{S_{X(2)}^2+S_{V(2)}^2}{\bar{X}^2} - 2\rho_{(2)} C_{Y(2)} C_{X(2)} \left\{1-\frac{1}{2}\mu\right\} \geq 0. \quad (5.3.3b)$$

From (5.3.3a), we have

$$\rho \leq \left\{1+\frac{1}{4}\mu\right\} \frac{C_X}{C_Y} \left(1+\frac{S_V^2}{S_X^2}\right), \quad (5.3.4a)$$

and from (5.3.3b), we have

$$\rho_{(2)} \leq \frac{1}{2}\left\{1+\frac{1}{2}\mu\right\} \frac{C_{X(2)}}{C_{Y(2)}} \left(1+\frac{S_{V(2)}^2}{S_{X(2)}^2}\right). \quad (5.3.4b)$$

6. NUMERICAL STUDY

We were unable to find real life data from any population. Due to this limitation and for the sake of empirical comparison of proposed estimator $t_{n(opt)}$ with Hansen and Hurwitz (1946) estimator $\bar{y}^*$, Cochran (1977) ratio estimator t_C and Singh and Kumar (2008) estimator t_{SK}, we generated artificial populations. Hansen and Hurwitz (1946), Cochran (1977) and Singh and Kumar (2008) have not considered measurement error while suggesting their estimators. For comparison purpose we have derived their MSE and provided above. It is obvious that increasing the nonresponse rate and decreasing size of re-contracting sample from non-respondents results in increase in MSE of an estimator. So we only considered one case of nonresponse rate and re-contracting sample size. We have generated six populations in which the correlation coefficient is low, high, positive and negative. The population size for each population is assumed to be 5000; respondent's population size is 3750 and sample size 500. It is assumed that 70% are respondents and further rec-contacting sample size is assumed to be 50% of non-respondents (30%) of main sample. The true auxiliary variable is assumed such that $X \sim N(10,2)$. Further the measured auxiliary variable is assumed such that $x = X + N(0,1)$ then $V = x - X$. The study variable is then generated by a linear model

$Y = 2 + bX + N(0,1)$. The value of b is changed to control the correlation between study and auxiliary variable. The assumed values are 0.1, 0.3, 0.5, -0.1, -.03 and -0.5. Similar to auxiliary variable the measured study variable is generated by $y = Y + N(0,1)$ then $U = y - Y$. The required parameters for MSE's and values of MSE of proposed and other three estimators are given in the following table 1. The last three rows of the table 1 contains percent relative efficiency (PRE) of Cochran (1977), Singh and Kumar (2008) and Proposed estimator w.r.t Hansen and Hurwitz (1946).

Table 1
Parameters of Population and Resulting MSEs and PRE of Estimators

Parameters	Pop-1	Pop-2	Pop-3	Pop-4	Pop-5	Pop-6
$\bar{Y}$	3.006713	5.011175	6.978384	1.007391	-0.9728	-2.95565
$\bar{X}$	10.06668	10.01243	9.981716	9.953722	9.950764	9.947114
S_Y^2	1.057322	1.384635	2.086024	1.031078	1.331736	1.968277
S_X^2	4.02115	4.065187	4.160332	4.062969	4.062725	4.029011
S_U^2	0.984729	0.953573	0.990767	1.044584	1.003714	0.999667
S_V^2	0.992094	0.991999	0.977486	1.040331	1.012271	1.038521
ρ_{YX}	0.163204	0.521635	0.712834	-0.18275	-0.50172	-0.71559
$S_{Y(2)}^2$	0.989206	1.323813	2.103051	1.022198	1.317754	2.015797
$S_{X(2)}^2$	4.030207	3.978491	4.042994	3.982557	4.178597	3.93733
$S_{U(2)}^2$	1.037355	0.932577	0.955694	1.015975	1.019975	0.934069
$S_{V(2)}^2$	0.996139	0.964786	0.925203	0.97084	1.044442	0.979257
$\rho_{YX(2)}$	0.123758	0.522869	0.703766	-0.16705	-0.51792	-0.71873
$MSE\left(\bar{y}^*\right)$	*0.00863*	*0.009724*	*0.013015*	*0.008718*	*0.009918*	*0.012553*
$MSE\left(t_C\right)$	*0.009806*	*0.009871*	*0.011172*	*0.009428*	*0.009038*	*0.009253*
$MSE\left(t_{SK}\right)$	*0.011859*	*0.014482*	*0.019447*	*0.009846*	*0.00889*	*0.009514*
$MSE\left(t_{n(opt)}\right)$	*0.008568*	*0.008473*	*0.009378*	*0.008791*	*0.00866*	*0.009013*
$PRE\left(t_C\right)$	88%	87%	77%	92%	95%	93%
$PRE\left(t_{SK}\right)$	73%	60%	44%	88%	97%	91%
$PRE\left(t_{n(opt)}\right)$	101%	102%	92%	98%	100%	96%

We also conducted simulation study considering above six populations. For this purpose, we selected 5000 samples from each population and absolute bias, percent relative bias (PRB), empirical mean square error (EMSE) and empirical percent relative efficiency (EPRE) for each estimator are computed. The results are given in the following table 2.

Table 2
Absolute Bias, PRB and Empirical MSE and EPRE

Parameters	Pop-1	Pop-2	Pop-3	Pop-4	Pop-5	Pop-6
$Bias\left(\bar{y}^*\right)$	*0.007436*	*0.009486*	*0.002779*	*0.002143*	*0.000992*	*0.022557*
$Bias\left(t_C\right)$	*0.006989*	*0.005063*	*0.01113*	*0.000992*	*0.001921*	*0.019978*
$Bias\left(t_{SK}\right)$	*0.006666*	*0.00113*	*0.019063*	*0.004046*	*0.002818*	*0.017128*
$Bias\left(t_{n(opt)}\right)$	*0.007297*	*0.007293*	*0.007607*	*0.003878*	*0.003034*	*0.019546*
$PRB\left(t_C\right)$	106%	147%	67%	750%	387%	37%
$PRB\left(t_{SK}\right)$	112%	658%	39%	184%	264%	43%
$PRB\left(t_{n(opt)}\right)$	102%	102%	98%	192%	245%	38%
$EMSE\left(\bar{y}^*\right)$	*0.002258*	*0.002709*	*0.003346*	*0.00232*	*0.002624*	*0.003269*
$EMSE\left(t_C\right)$	*0.002425*	*0.002683*	*0.002759*	*0.002465*	*0.002405*	*0.002435*
$EMSE\left(t_{SK}\right)$	*0.003258*	*0.004592*	*0.005959*	*0.002657*	*0.002319*	*0.002578*
$EMSE\left(t_{n(opt)}\right)$	*0.002205*	*0.002364*	*0.002395*	*0.00229*	*0.002281*	*0.002389*
$EPRE\left(t_C\right)$	93%	84%	82%	92%	94%	93%
$EPRE\left(t_{SK}\right)$	69%	49%	38%	85%	97%	88%
$EPRE\left(t_{n(opt)}\right)$	102%	96%	94%	99%	99%	95%

7. CONCLUSION

On the basis of percent relative efficiency given in table 1, it is concluded that for all six populations the proposed estimator is better than other three estimators. Table 2 depicts that proposed estimator is less biased as compared to other two biased estimators. Also the proposed estimator has more empirical PRE than other three estimators.

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